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Problem Set 6

Due Thursday, April 30, in class

1. LR, Wald, and LM Statistics for a Bernoulli Sample

Let $X_i \sim iid$ Bernoulli(θ). Consider testing $H_0 : \theta = \theta_0$ vs. $H_1 : \theta \neq \theta_0$ using a 5% significance level.

- Derive the LR, Wald, and LM statistics.
- Compare the Wald and LM statistics. In particular, comment on how the information matrix is computed for the two statistics.
- Suppose $N = 50$ and $\overline{X}_N = 0.65$. Compute the trilogy of statistics, and use all three to test the null that $\theta = 0.5$. Report the results.

2. Suppose we have a sample of *n* iid Normal random variables,

$$X_i \sim iidN(\mu, \sigma^2),$$

where both the mean and variance are unknown. Let $\theta = (\mu, \sigma^2)'$.

- Write down the likelihood function for θ.
- Write down the log-likelihood function for θ.
- Derive the maximum likelihood estimators of μ and σ^2. Are they unbiased?
- Derive the information matrix for μ and σ^2.
- Does the maximum likelihood estimator of μ achieve the Cramér-Rao lower bound?
- Write down the LR, Wald, and LM test statistics for the null that $\mu = \mu_0$.

3. Consider the partitioned regression model

$$y_i = \gamma x_i + \beta z_i + u_i$$

with *iid* Gaussian errors. **Assume that the regressors have zero mean.**

- Derive the information matrix for $\hat{\gamma}$ and $\hat{\beta}$ (you can ignore the error variance, since the information matrix is block diagonal).
- Derive the estimated variances of $\hat{\gamma}$ and $\hat{\beta}$ in terms of sample moments of the data; (assume you have an estimator of $\hat{\sigma}^2$).
- Derive the squared *t*-statistic for the null that $\beta_0 = 0$.

4. Consider the following regression model with Normal errors:

$$Y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + u_i$$

where $u_i \sim iidN(0, \sigma^2)$, and $X_{1i} = 1$. Using the data in ps6.wf1, tests the following hypotheses using the LR, Wald, and LM tests.

$$H_0 : \beta_2 = 0, \beta_3 = 0$$
$$H_1 : \beta_2 \neq 0, \beta_3 \neq 0$$

5. Prove that in the model

$$y_1 = X_1 \beta_1 + u_1$$
$$y_2 = X_2 \beta_2 + u_2,$$

with freely correlated disturbances, that equation by equation OLS is equivalent to SUR if $X_1 = X_2$.

6. Consider the system

$$\begin{aligned} y_{1i} &= \alpha_1 + \gamma z_i + u_{1i} \\ y_{2i} &= \alpha_2 \qquad\quad + u_{2i}, \end{aligned}$$

where the disturbances are freely correlated. Prove that SUR leads to the OLS estimates of α_1 and α_2, but to a mixture of the OLS slope estimates from regressions of y_1 on z and y_2 on z respectively. What is this mixture? To simplify the algebra, assume that $\bar{z} = 0$, without loss of generality.

Hint:

$$\begin{bmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & d \end{bmatrix}^{-1} = \begin{bmatrix} \frac{d}{ad-b^2} & 0 & \frac{-b}{ad-b^2} \\ 0 & \frac{1}{c} & 0 \\ \frac{-b}{ad-b^2} & 0 & \frac{a}{ad-b^2} \end{bmatrix}.$$