

### Problem Set 5

Due Thursday, April 14, in class

1. Suppose you have data which are *iid* Poisson( $\lambda$ ). That is,

$$f(x_i) = \frac{e^{-\lambda} \lambda^{x_i}}{x_i!}$$

where  $x = 0, 1, \dots$ ,  $E(X_i) = \lambda$ , and  $\text{var}(X_i) = \lambda$ .

- Derive the log likelihood function.
- Find the maximum likelihood estimator of  $\lambda$ , denoted  $\hat{\lambda}_{MLE}$ .
- Derive the Cramér-Rao lower bound for unbiased estimators of  $\lambda$ . Does  $\hat{\lambda}_{MLE}$  achieve the Cramér-Rao lower bound?

2. Suppose you have data, which are *iid* exponential. That is

$$f(z_i; \theta) = \frac{1}{\theta} e^{-z_i/\theta}$$

where  $0 \leq z_i < \infty$ , and  $\theta > 0$ . Also,  $E(z_i) = \theta$  and  $\text{var}(z_i) = \theta^2$ .

- Derive the log likelihood function.
- Derive the maximum likelihood estimator of  $\theta$ ,  $\hat{\theta}_{MLE}$ . Is  $\hat{\theta}_{MLE}$  unbiased?
- Derive the Cramér-Rao lower bound for unbiased estimators of  $\theta$ . Does  $\hat{\theta}_{MLE}$  achieve the Cramér-Rao lower bound?

3. Consider the regression model  $y = X\beta + u$ , where  $u \sim (0, \sigma^2 \Sigma)$  and  $\Sigma$  is known.

Prove that  $\sigma_{GLS}^2$  is unbiased, and  $\sigma_{OLS}^2$  is biased.

4. Redo question 3 from the midterm, in Eviews, using midterm.wf1.