Econ 7331
Econometrics 1
Spring 2009

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Problem Set 3

Due date: Thursday, February 26, in class**.**

1. Consider two linear regressions, one restricted and the other unrestricted:

$$y = X\beta + u$$

$$y = X\beta + Z\gamma + u\,.$$

The former is the restricted model, with the restriction $\gamma = 0$ imposed.

Show that, in the case of mutually orthogonal regressors, *i.e.* $X'Z = 0$, the estimator for β is identical for each regression.

Consider minimization of the sum of squared residuals, subject to the restriction that a set of linear hypotheses hold, *i.e.* $R\beta - r = 0$. Call the restricted least squares estimator $\tilde{\beta}$.

2. Show that $\tilde{\beta} = \hat{\beta} + (X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}(r - R\hat{\beta})$.

3. Show that the covariance matrix of $\tilde{\beta}$ is:

$$\text{cov}(\tilde{\beta}) = \sigma^2 M^*(X'X)^{-1}M^{*\prime}$$

where $M^* = I - (X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}R$.

4. Since M^* is idempotent, but not symmetric, show that the covariance formula reduces to:

$$\text{cov}(\tilde{\beta}) = \sigma^2 M^*(X'X)^{-1}$$

5. Consider the following regression model with Normal errors:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

where $u_i \sim iidN(0, \sigma^2)$.

a. Construct a 95% confidence interval for β_2. Use this confidence interval to test the following hypothesis:

$$H_0 : \beta_2 = 2.3$$
$$H_1 : \beta_2 \neq 2.3$$

b. Test the following hypothesis using a *t*-statistic:

$$H_0 : \beta_2 + \beta_3 = 5$$
$$H_1 : \beta_2 + \beta_3 \neq 5$$

c. Test the following hypotheses:

$$H_0 : \beta_0 = 0, \beta_3 = 0$$
$$H_1 : \beta_0 \neq 0, \beta_3 \neq 0$$

d. Using a p-value, test the null that $\beta_0 = 0$ against the 2-sided alternative.

Regression Output for Problem 5

Dependent Variable: Y
Method: Least Squares
Date: 03/12/08 Time: 15:02
Sample: 1 15
Included observations: 15

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.445737	0.368727	1.208854	0.2521
X1	0.892575	0.171089	5.217019	0.0003
X2	2.361413	0.724042	3.261432	0.0076
X3	3.155499	0.149056	21.16995	0.0000

R-squared	0.995800	Mean dependent var	-6.154827
Adjusted R-squared	0.994654	S.D. dependent var	18.11850
S.E. of regression	1.324742	Akaike info crit	3.623492
Sum squared resid	19.30437	Schwarz criterion	3.812305
Log likelihood	-23.17619	F-statistic	869.2809
Durbin-Watson stat	1.483394	Prob(F-statistic)	0.000000

Coefficient Covariance Matrix

	C	X1	X2	X3
C	0.135960	0.010335	-0.003392	0.002504
X1	0.010335	0.029271	-0.026940	-0.019129
X2	-0.003392	-0.026940	0.524237	-0.023940
X3	0.002504	-0.019129	-0.023940	0.022218

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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 03/12/08   Time: 16:31
Sample: 1 15
Included observations: 15
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C          -6.154827   4.678176  -1.315647   0.2094
============================================================
R-squared            0.000000    Mean dependent var-6.154827
Adjusted R-squared   0.000000    S.D. dependent var 18.11850
S.E. of regression   18.11850    Akaike info crit   8.696084
Sum squared resid    4595.919    Schwarz criterion  8.743288
Log likelihood      -64.22063    Durbin-Watson stat 2.336955
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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 03/12/08   Time: 16:31
Sample: 1 15
Included observations: 15
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           0.090116   2.278498   0.039550   0.9691
        X1           3.609472   0.699869   5.157357   0.0002
        X2           5.761485   4.367186   1.319267   0.2117
============================================================
R-squared            0.824668    Mean dependent var-6.154827
Adjusted R-squared   0.795446    S.D. dependent var 18.11850
S.E. of regression   8.194568    Akaike info crit   7.221677
Sum squared resid    805.8114    Schwarz criterion  7.363287
Log likelihood      -51.16257    F-statistic        28.22080
Durbin-Watson stat   2.573820    Prob(F-statistic)  0.000029
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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 03/12/08   Time: 16:31
Sample: 1 15
Included observations: 15
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
        X1           3.601176   0.641539   5.613342   0.0001
        X2           5.761946   4.196115   1.373162   0.1929
============================================================
R-squared            0.824645    Mean dependent var-6.154827
Adjusted R-squared   0.811156    S.D. dependent var 18.11850
S.E. of regression   7.873600    Akaike info crit   7.088474
Sum squared resid    805.9165    Schwarz criterion  7.182880
Log likelihood      -51.16355    Durbin-Watson stat 2.575678
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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 03/12/08   Time: 16:31
Sample: 1 15
Included observations: 15
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
        X1           0.979464   0.220030   4.451504   0.0007
        X3           3.255368   0.193977   16.78224   0.0000
============================================================
R-squared            0.991141    Mean dependent var-6.154827
Adjusted R-squared   0.990460    S.D. dependent var 18.11850
S.E. of regression   1.769733    Akaike info crit   4.103100
Sum squared resid    40.71540    Schwarz criterion  4.197507
Log likelihood      -28.77325    Durbin-Watson stat 2.179030
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Dependent Variable: Y
Method: Least Squares
Date: 03/12/08 Time: 16:31
Sample: 1 15
Included observations: 15

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.461016	0.495082	0.931191	0.3701
X1	1.013924	0.224238	4.521646	0.0007
X3	3.263335	0.195164	16.72102	0.0000

R-squared	0.991738	Mean dependent var	-6.154827
Adjusted R-squared	0.990361	S.D. dependent var	18.11850
S.E. of regression	1.778848	Akaike info crit	4.166665
Sum squared resid	37.97159	Schwarz criterion	4.308275
Log likelihood	-28.24999	F-statistic	720.2144
Durbin-Watson stat	2.269623	Prob(F-statistic)	0.000000