

Problem Set 3

Due date: Tuesday, March 2, in class.

1. Consider two linear regressions, one restricted and the other unrestricted:

$$y = X\beta + u$$

$$y = X\beta + Z\gamma + u.$$

The former is the restricted model, with the restriction $\gamma = 0$ imposed.

Show that, in the case of mutually orthogonal regressors, *i.e.* $X'Z = 0$, the estimator for β is identical for each regression.

Consider minimization of the sum of squared residuals, subject to the restriction that a set of linear hypotheses hold, *i.e.* $R\beta - r = 0$. Call the restricted least squares estimator $\tilde{\beta}$.

2. Show that $\tilde{\beta} = \hat{\beta} + (X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}(r - R\hat{\beta})$.

3. Show that the covariance matrix of $\tilde{\beta}$ is:

$$\text{cov}(\tilde{\beta}) = \sigma^2 M^* (X'X)^{-1} M^{*'} ,$$

where $M^* = I - (X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}R$.

4. Since M^* is idempotent, but not symmetric, show that the covariance formula reduces to:

$$\text{cov}(\tilde{\beta}) = \sigma^2 M^* (X'X)^{-1}$$

5. Consider the following regression model with Normal errors:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

where $u_i \sim iidN(0, \sigma^2)$.

a. Construct a 95% confidence interval for β_2 . Use this confidence interval to test the following hypothesis:

$$H_0 : \beta_2 = 2.3$$

$$H_1 : \beta_2 \neq 2.3$$

b. Test the following hypothesis using a t -statistic:

$$H_0 : \beta_2 + \beta_3 = 5$$

$$H_1 : \beta_2 + \beta_3 \neq 5$$

c. Test the following hypotheses:

$$H_0 : \beta_0 = 0, \beta_3 = 0$$

$$H_1 : \beta_0 \neq 0, \beta_3 \neq 0$$

d. Using a p-value, test the null that $\beta_0 = 0$ against the 2-sided alternative.

Regression Output for Problem 5

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Dependent Variable: Y
Method: Least Squares
Date: 03/12/08   Time: 15:02
Sample: 1 15
Included observations: 15
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Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.445737	0.368727	1.208854	0.2521
X1	0.892575	0.171089	5.217019	0.0003
X2	2.361413	0.724042	3.261432	0.0076
X3	3.155499	0.149056	21.16995	0.0000

R-squared	0.995800	Mean dependent var	-6.154827
Adjusted R-squared	0.994654	S.D. dependent var	18.11850
S.E. of regression	1.324742	Akaike info crit	3.623492
Sum squared resid	19.30437	Schwarz criterion	3.812305
Log likelihood	-23.17619	F-statistic	869.2809
Durbin-Watson stat	1.483394	Prob(F-statistic)	0.000000

Coefficient Covariance Matrix

	C	X1	X2	X3
C	0.135960	0.010335	-0.003392	0.002504
X1	0.010335	0.029271	-0.026940	-0.019129
X2	-0.003392	-0.026940	0.524237	-0.023940
X3	0.002504	-0.019129	-0.023940	0.022218

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=====
Dependent Variable: Y
Method: Least Squares
Date: 03/12/08   Time: 16:31
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=====

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Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-6.154827	4.678176	-1.315647	0.2094
R-squared	0.000000	Mean dependent var	-6.154827	
Adjusted R-squared	0.000000	S.D. dependent var	18.11850	
S.E. of regression	18.11850	Akaike info crit	8.696084	
Sum squared resid	4595.919	Schwarz criterion	8.743288	
Log likelihood	-64.22063	Durbin-Watson stat	2.336955	

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=====

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Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.090116	2.278498	0.039550	0.9691
X1	3.609472	0.699869	5.157357	0.0002
X2	5.761485	4.367186	1.319267	0.2117
R-squared	0.824668	Mean dependent var	-6.154827	
Adjusted R-squared	0.795446	S.D. dependent var	18.11850	
S.E. of regression	8.194568	Akaike info crit	7.221677	
Sum squared resid	805.8114	Schwarz criterion	7.363287	
Log likelihood	-51.16257	F-statistic	28.22080	
Durbin-Watson stat	2.573820	Prob(F-statistic)	0.000029	

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=====

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Variable	Coefficient	Std. Error	t-Statistic	Prob.
X1	3.601176	0.641539	5.613342	0.0001
X2	5.761946	4.196115	1.373162	0.1929
R-squared	0.824645	Mean dependent var	-6.154827	
Adjusted R-squared	0.811156	S.D. dependent var	18.11850	
S.E. of regression	7.873600	Akaike info crit	7.088474	
Sum squared resid	805.9165	Schwarz criterion	7.182880	
Log likelihood	-51.16355	Durbin-Watson stat	2.575678	

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Variable	Coefficient	Std. Error	t-Statistic	Prob.
X1	0.979464	0.220030	4.451504	0.0007
X3	3.255368	0.193977	16.78224	0.0000
R-squared	0.991141	Mean dependent var	-6.154827	
Adjusted R-squared	0.990460	S.D. dependent var	18.11850	
S.E. of regression	1.769733	Akaike info crit	4.103100	
Sum squared resid	40.71540	Schwarz criterion	4.197507	
Log likelihood	-28.77325	Durbin-Watson stat	2.179030	

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Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.461016	0.495082	0.931191	0.3701
X1	1.013924	0.224238	4.521646	0.0007
X3	3.263335	0.195164	16.72102	0.0000

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R-squared          0.991738      Mean dependent var-6.154827
Adjusted R-squared 0.990361      S.D. dependent var 18.11850
S.E. of regression 1.778848      Akaike info crit  4.166665
Sum squared resid  37.97159      Schwarz criterion  4.308275
Log likelihood      -28.24999      F-statistic        720.2144
Durbin-Watson stat 2.269623      Prob(F-statistic) 0.000000
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