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Problem Set 2

Due date: Thursday, February 5, in class.

Consider the simple linear regression model with one regressor:

$$y_i = \beta x_i + u_i,$$

where y_i and x_i are expressed in deviations from their means. Assume that the standard classical assumptions hold. Specifically, the $x_i's$ are fixed in repeated sampling, and $u_i \sim iid(0,\sigma^2)$.

1. Consider the following linear estimator of β:

$$\tilde{\beta} = \frac{1}{N}\sum_{i=1}^{N}\frac{y_i}{x_i}$$

Derive the mean and variance of $\tilde{\beta}$. Compare the variance of $\tilde{\beta}$ to $\frac{\sigma^2}{\sum x_i^2}$. Which is larger? Are the residuals, $y_i - \tilde{\beta}x_i$, uncorrelated with the explanatory variables?

2. Answer the same question with $\tilde{\beta} = \frac{y_2 - y_1}{x_2 - x_1}$.

3. Consider the following regression model $y_i = \beta x_i + u_i$, with $u_i \sim iidN(0,\sigma^2)$, where $i = 1, 2, \ldots, 10$;

Suppose that $\sum x_i y_i = 17900$, $\sum x_i^2 = 39400$, and $\sum \hat{u}_i^2 = 283.27$

Consider the following hypothesis and 2-sided alternative:

$$\begin{aligned} H_0 &: \beta = 0.50 \\ H_1 &: \beta \neq 0.50 \end{aligned}.$$

Test the above hypothesis using a confidence interval and the test of significance approach. Set the size of your test to 0.05 and 0.01. Report the outcome of both tests.

4. Prove the Frisch-Waugh Theorem. That is, in the linear regression model

$$y = X_1\beta_1 + X_2\beta_2 + u,$$

demonstrate that

$$\hat{\beta}_1 = \left(X_1^{*\prime} X_1^{*}\right)^{-1} X_1^{*\prime} y^{*},$$

where $X_1^* \equiv M_2 X_1$, $y^* \equiv M_2 y$, and $M_2 = I - X_2(X_2'X_2)^{-1}X_2'$.

5. Consider a nonsingular linear transformation of the regressors, XA, where the matrix A is a k x k and invertible.

Show that the fitted values and the residuals from a regression of y on XA are the same as from a regression of y on X.

The following results from matrix algebra will be helpful:

$(BC)' = C'B'$

$(BC)^{-1} = C^{-1}B^{-1}$, whenever all three inverses exist.