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Econometrics 1 McElhinney 211 D
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Problem Set 1

Due date: Thursday, January 30, in class.

1. Consider the linear regression model $Y_i = \alpha + \beta X_i + u_i$. The OLS estimators of α and β are as follows:

$$\hat{\beta} = \frac{N\sum X_i Y_i - \sum X_i \sum Y_i}{N\sum X_i^2 - \left(\sum X_i\right)^2} \tag{1}$$

$$\hat{\alpha} = \frac{\sum X_i^2 \sum Y_i - \sum X_i \sum X_i Y_i}{N\sum X_i^2 - \left(\sum X_i\right)^2}. \tag{2}$$

Demonstrate that $\hat{\alpha}$ and $\hat{\beta}$ can be more compactly written as:

$$\hat{\beta} = \frac{\sum x_i y_i}{\sum x_i^2} \tag{1'}$$

$$\hat{\alpha} = \overline{Y} - \hat{\beta}\overline{X} \tag{2'}$$

where $x_i \equiv X_i - \overline{X}$ and $y_i \equiv Y_i - \overline{Y}$.

Now demonstrate that if you regressed y_i on x_i, the estimator for β would be the same as in equation (1)'.

This result is a rudimentary form of the Frisch-Waugh theorem, which we will prove more generally later. In this context, the Frisch-Waugh theorem states that there are two equivalent methods of obtaining $\hat{\beta}$:

- Regress Y_i on a constant and X_i. $\hat{\beta}$ is then the regression coefficient on X_i.
- Remove the mean from both Y_i and X_i then regress y_i on x_i. $\hat{\beta}$ is then the regression coefficient on X_i.

2. Prove the following:

$$E(\hat{\alpha}) = \alpha$$

$$\text{var}(\hat{\alpha}) = \frac{\sigma^2 \sum X_i^2}{N \sum x_i^2}.$$

$$\text{cov}(\hat{\alpha}, \hat{\beta}) = \frac{-\bar{X}\sigma^2}{\sum x_i^2}$$

3. Prove that if Y_i is regressed only on a constant that the OLS estimator is equal to the sample mean of the dependent variable. What is the R^2 from this regression?

4. Consider a regression without a constant term, *i.e.* $Y_i = \beta X_i + u_i$. Y_i and X_i are not in deviations from their respective means. Argue that the R^2 from this regression *may* be negative.