Econ 7331 | Professor Chris Murray
Econometrics 1 | McElhinney 211 D
Spring 2008

Final Exam

Write your answers on the blank paper that I have given you. Do not write your answers on this exam. **You must explain your answers.** If you are confused about a question or you think it is unclear, please ask for clarification before answering**. When testing a hypothesis, make sure to write down the null and alternative, the critical value(s), the test statistic, and your decision (reject or fail to reject).** Use tests with 5% size. Unless otherwise specified, use 2-sided alternative hypotheses.

1. Consider the linear regression model with Gaussian errors

$$y = X\beta + u \; ; \; u \sim N(0, \sigma^2 I) .$$

Let $\theta = (\beta', \sigma^2)'$ denote the $(k+1)$ x 1 vector of parameters.

a. Construct the Gaussian likelihood function for θ. **(5 points)**

b. Setting the Score to zero, solve for the Maximum Likelihood Estimators of β and σ^2. Are $\hat{\beta}_{MLE}$ and $\hat{\sigma}^2{}_{MLE}$ unbiased? **(10 points)**

c. Derive the Information Matrix, $I(\theta)$ and its inverse, $I(\theta)^{-1}$.**(10 points)**

d. Discuss the optimality or lack thereof of $\hat{\beta}_{MLE}$ and $\hat{\sigma}^2{}_{MLE}$. Be explicit. **(10 points)**

e. Consider testing the following multiple linear hypotheses:

$$H_0 : R\beta - r = 0$$
$$H_1 : R\beta - r \neq 0 .$$

Prove that the Lagrange Multiplier statistic for this set of hypotheses is equal to the sample size multiplied by the uncentered R^2 from a regression of u_0 on X, where u_0 is the vector of restricted residuals: $u_0 = y - X\beta_0$, where β_0 satisfies the hypothesized restrictions. **(15 points)**

f. Now assume that the errors are not Gaussian, *i.e.* $u \sim (0, \sigma^2 I)$. Discuss the optimality of the estimator for β derived in part a. **(10 points)**

2. Consider the partitioned regression model:

$$y = X_1\beta_1 + X_2\beta_2 + u \, .$$

a. Prove and **interpret** the Frisch-Waugh result that

$\hat{\beta}_1 = (X_1' M_2 X_1)^{-1} X_1' M_2 y$, where $M_2 = I - X_2(X_2' X_2)^{-1} X_2'$. **(10 points)**

b. Suppose a time series of interest is thought to be a stable (or stationary) 1st order autoregression, or AR(1), around a deterministic quadratic trend:

$$y_t = a + bt + ct^2 + \phi y_{t-1} + u_t \, ,$$

where $|\phi| < 1$, and $u_t \sim iid(0, \sigma^2)$.

Give a Frisch-Waugh interpretation to $\hat{\phi}$ when ϕ is estimated from a partitioned regression. In this case, $X_2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ \vdots & \vdots & \vdots \\ 1 & T & T^2 \end{bmatrix}$, and T is the number of observations.

(10 points)

3. Consider testing $H_0 : \theta = \theta_0$ vs. $H_1 : \theta \neq \theta_0$ in a Maximum Likelihood Context

Discuss how the Likelihood Ratio test accounts for both the curvature of the likelihood function, as well as the difference between the estimated value and hypothesized value of θ.

Explain how the Wald test and the Lagrange Multiplier test take the curvature of the log-likelihood function into account when assessing whether or not there is evidence against the null hypothesis. **(20 points)**

4. Consider the following regression model:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i,$$

where $u_i \sim iidN(0, \sigma^2)$.

Use the regression output on the following pages to test the hypotheses below. Use a likelihood based test for parts a - c. The dependent variable "u" is the residual from the most recent regression with "y" as the dependent variable.

a. $$H_0: \beta_1 = 0, \beta_2 = 0$$ $$H_1: \beta_1 \neq 0, \beta_2 \neq 0$$ **(10 points)**

b. $$H_0: \beta_1 = 0$$ $$H_1: \beta_1 \neq 0$$ **(10 points)**

c. $$H_0: \beta_0 = 0$$ $$H_1: \beta_0 \neq 0$$ **(10 points)**

d. Use a 95% confidence interval to test that null that $\beta_2 + \beta_3 = 1$ against the 2-sided alternative. **(10 points)**

e. Test the null hypothesis that $\beta_2^3 = 1$ against the 2-sided alternative. **(10 points)**

Regression Output for Question 4

Dependent Variable: Y
Method: Least Squares
Date: 04/29/08 Time: 17:54
Sample: 1 15
Included observations: 15

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.337212			
X1	1.677726			
X2	2.084239	2.172125	0.959539	0.3579
X3	0.466498	0.447167	1.043231	0.3192

R-squared	0.872580	Mean dependent var	0.790991
Adjusted R-squared	0.837829	S.D. dependent var	9.868840
S.E. of regression	3.974227	Akaike info criteri	5.820716
Sum squared resid	173.7393	Schwarz criterion	6.009530
Log likelihood	-39.65537	F-statistic	25.10954
Durbin-Watson stat	1.483394	Prob(F-statistic)	0.000032

Coefficient Covariance Matrix

	C	X1	X2	X3
C			-0.030527	0.022535
X1			-0.242457	-0.172165
X2	-0.030527	-0.242457	4.718129	-0.215456
X3	0.022535	-0.172165	-0.215456	0.199958

Dependent Variable: Y
Method: Least Squares
Date: 04/29/08 Time: 17:56
Sample: 1 15
Included observations: 15

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.284638	1.109093	3.863192	0.0023
X1	2.079383	0.340671	6.103777	0.0001
X2	2.586894	2.125793	1.216908	0.2470

R-squared	0.859973	Mean dependent var	0.790991
Adjusted R-squared	0.836635	S.D. dependent var	9.868840
S.E. of regression	3.988828	Akaike info criteri	5.781728
Sum squared resid	190.9290	Schwarz criterion	5.923338
Log likelihood	-40.36296	F-statistic	36.84891
Durbin-Watson stat	1.429938	Prob(F-statistic)	0.000008

Dependent Variable: Y
Method: Least Squares
Date: 04/29/08 Time: 18:00
Sample: 1 15
Included observations: 15

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.700360	1.501094	2.465109	0.0284
X3	1.864734	0.326613	5.709313	0.0001

R-squared	0.714889	Mean dependent var	0.790991
Adjusted R-squared	0.692957	S.D. dependent var	9.868840
S.E. of regression	5.468466	Akaike info criteri	6.359439
Sum squared resid	388.7536	Schwarz criterion	6.453846
Log likelihood		F-statistic	32.59626
Durbin-Watson stat	2.045364	Prob(F-statistic)	0.000072

Dependent Variable: Y
Method: Least Squares
Date: 04/29/08 Time: 17:56
Sample: 1 15
Included observations: 15

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.350697	1.102432	3.946453	0.0019
X1	1.784832	0.499325	3.574487	0.0038
X3	0.561676	0.434584	1.292447	0.2205

R-squared	0.861915	Mean dependent var	0.790991
Adjusted R-squared	0.838900	S.D. dependent var	9.868840
S.E. of regression	3.961077	Akaike info criteri	5.767765
Sum squared resid	188.2815	Schwarz criterion	5.909375
Log likelihood	-40.25824	F-statistic	37.45140
Durbin-Watson stat	1.737211	Prob(F-statistic)	0.000007

Dependent Variable: Y
Method: Least Squares
Date: 04/29/08   Time: 17:56
Sample: 1 15
Included observations: 15

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.744822	1.466906	2.552871	0.0253
X2	3.628313	2.850023	1.273082	0.2271
X3	1.562920	0.397515	3.931730	0.0020

R-squared		Mean dependent var	0.790991
Adjusted R-squared	0.706950	S.D. dependent var	9.868840
S.E. of regression	5.342406	Akaike info criteri	6.366086
Sum squared resid		Schwarz criterion	6.507696
Log likelihood	-44.74564	F-statistic	17.88672
Durbin-Watson stat	1.588090	Prob(F-statistic)	0.000251

Dependent Variable: U
Method: Least Squares
Date: 04/29/08   Time: 17:57
Sample: 1 15
Included observations: 15

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.46E-15	1.102432	1.33E-15	1.0000
X1	2.81E-15	0.499325	5.63E-15	1.0000
X3	-1.85E-15	0.434584	-4.26E-15	1.0000

R-squared	0.000000	Mean dependent var	-1.65E-33
Adjusted R-squared	-0.166667	S.D. dependent var	3.667244
S.E. of regression	3.961077	Akaike info criteri	5.767765
Sum squared resid	188.2815	Schwarz criterion	5.909375
Log likelihood	-40.25824	Durbin-Watson stat	1.737211

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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 04/29/08   Time: 17:57
Sample: 1 15
Included observations: 15
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
        X1          1.348016   0.750633   1.795840   0.0977
        X2          2.192443   3.219896   0.680905   0.5089
        X3          0.386622   0.662232   0.583817   0.5702
============================================================
R-squared                        Mean dependent var 0.790991
Adjusted R-squared  0.643584     S.D. dependent var 9.868840
S.E. of regression  5.891756     Akaike info criteri6.561842
Sum squared resid                Schwarz criterion  6.703452
Log likelihood                   Durbin-Watson stat 0.670880
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============================================================
Dependent Variable: U
Method: Least Squares
Date: 04/29/08   Time: 17:57
Sample: 1 15
Included observations: 15
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
        X1          1.53E-15   0.750633   2.04E-15   1.0000
        X2         -2.28E-15   3.219896  -7.09E-16   1.0000
        X3         -7.88E-16   0.662232  -1.19E-15   1.0000
============================================================
R-squared          -1.006453     Mean dependent var 3.732261
Adjusted R-squared -1.340862     S.D. dependent var 3.850852
S.E. of regression  5.891756     Akaike info criteri6.561842
Sum squared resid   416.5534     Schwarz criterion  6.703452
Log likelihood     -46.21381     Durbin-Watson stat 0.670880
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============================================================
Dependent Variable: U
Method: Least Squares
Date: 04/29/08   Time: 17:57
Sample: 1 15
Included observations: 15
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C          4.337212   1.106182   3.920886   0.0024
        X1          0.329710   0.513267   0.642374   0.5338
        X2         -0.108204   2.172125  -0.049815   0.9612
        X3          0.079876   0.447167   0.178627   0.8615
============================================================
R-squared           0.163133     Mean dependent var 3.732261
Adjusted R-squared -0.065104     S.D. dependent var 3.850852
S.E. of regression  3.974227     Akaike info criteri5.820716
Sum squared resid   173.7393     Schwarz criterion  6.009530
Log likelihood     -39.65537     F-statistic        0.714754
Durbin-Watson stat  1.483394     Prob(F-statistic)  0.563428
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