Econ 7331 Professor Chris Murray
Econometrics 1 McElhinney 211 D
Spring 2007

Final Exam

Write your answers on one side of the blank white paper that I have given you.. Do not write your answers on this exam. **You must explain your answers.** If you are confused about a question or you think it is unclear, please ask for clarification before answering**. When testing a hypothesis, make sure to write down the null and alternative, the critical value(s), the test statistic, and your decision (reject or fail to reject).** Use tests with 5% size. Unless otherwise specified, use 2-sided alternative hypotheses.

1. Consider the linear regression model with Gaussian errors

$$y = X\beta + u \; ; \; u \sim N(0, \sigma^2 I) \, .$$

Let $\theta = (\beta', \sigma^2)'$ denote the $(k+1)$ x 1 vector of parameters.

a. Prove that the Maximum Likelihood Estimator of β is equal to the Ordinary Least Squares estimator of β. **(10 points)**

b. Discuss the optimality of $\hat{\beta}_{MLE}$. Be explicit. **(10 points)**

c. Derive the Maximum Likelihood Estimator of σ^2. Is it optimal? **(10 points)**

d. Consider testing the following multiple linear hypotheses:

$$H_0 : R\beta - r = 0$$
$$H_1 : R\beta - r \neq 0 \, .$$

Prove that the Lagrange Multiplier statistic for this set of hypotheses is equal to the sample size multiplied by the uncentered R^2 from a regression of u_0 on X, where u_0 is the vector of restricted residuals: $u_0 = y - X\beta_0$, where β_0 satisfies the hypothesized restrictions. **(20 points)**

e. Relax the assumption of Gaussian errors, *i.e.* $u \sim (0, \sigma^2 I)$. Now discuss the optimality of the estimator for β derived in part a. **(10 points)**

2. Consider the linear regression model with Gaussian errors

$$y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$$

where $u_i \sim iidN(0, \sigma^2)$.

Use the regression output below to tests the following hypotheses against their 2-sided alternatives.

a. $H_0 : \beta_1 + \beta_2 = 9$ **(10 points)**

b. $H_0 : \beta_1^2 + \beta_2^{1/3} = 3$, and $\beta_1^4 = 1$ **(20 points)**

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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 05/06/07   Time: 12:53
Sample: 1 20
Included observations: 20
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
        X1           0.826637   0.382180   2.162953   0.0442
        X2           10.08943   2.554858   3.949115   0.0009
============================================================
R-squared            0.739383    Mean dependent var-1.074176
Adjusted R-squared   0.724905    S.D. dependent var 9.736475
S.E. of regression   5.106736    Akaike info criteri6.193637
Sum squared resid    469.4175    Schwarz criterion  6.293211
Log likelihood      -59.93637    Durbin-Watson stat 2.297714
============================================================


   Coefficient Covariance Matrix
====================================
                 X1          X2
====================================
     X1       0.146061   -0.618631
     X2      -0.618631    6.527297
====================================
```

3. Consider the partitioned regression model:

$$y = X_1\beta_1 + X_2\beta_2 + u\,.$$

a. Prove and interpret the Frisch-Waugh result that

$\hat{\beta}_1 = (X_1'M_2X_1)^{-1}X_1'M_2y$, where $M_2 = I - X_2(X_2'X_2)^{-1}X_2'$. **(20 points)**

b. Interpret $\hat{\rho}$ in the Frisch-Waugh context when the regression model is

$$y_t = a + bt + \rho y_{t-1} + u_t\,.$$ **(10 points).**

4. Consider the following Gaussian regression model:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i, \text{ where } u_i \sim iidN(0, \sigma^2)$$

Use the regression output on the following pages to test the hypotheses below. Use a likelihood based hypothesis test: LR, Wald, or LM. Use the LM test whenever possible. In every case, the dependent variable "u" is the residual from the most recent regression with "y" as the dependent variable.

a. $$H_0 : \beta_1 = 0, \beta_2 = 0, \beta_3 = 0$$
$$H_1 : \beta_1 \neq 0, \beta_2 \neq 0, \beta_3 \neq 0$$
(10 points)

b. $$H_0 : \beta_1 = 0, \beta_2 = 0$$
$$H_1 : \beta_1 \neq 0, \beta_2 \neq 0$$
(10 points)

c. What would be the result of your test if you performed a LM test in part b? What about a Wald test? **(10 points)**

d. $$H_0 : \beta_2 = 0, \beta_3 = 0$$
$$H_1 : \beta_2 \neq 0, \beta_3 \neq 0$$
(10 points)

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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 05/06/07   Time: 14:58
Sample: 1 18
Included observations: 18
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           0.640979   1.404188   0.456477   0.6550
        X1           0.730066   0.703965   1.037076   0.3173
        X2           3.853749   3.099397   1.243387   0.2341
        X3           3.168290   0.598357   5.294979   0.0001
============================================================
R-squared            0.929280    Mean dependent var-3.840417
Adjusted R-squared   0.914126    S.D. dependent var 19.54705
S.E. of regression   5.728111    Akaike info criteri6.521779
Sum squared resid    459.3576    Schwarz criterion  6.719639
Log likelihood      -54.69601    F-statistic
Durbin-Watson stat   2.317055    Prob(F-statistic)
============================================================
```

```
============================================================
Dependent Variable: Y
Method: Least Squares
Date: 05/06/07   Time: 14:59
Sample: 1 18
Included observations: 18
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           0.709369   2.350586   0.301784   0.7670
        X1           3.582936   0.758468   4.723910   0.0003
        X2           7.125708   5.084388   1.401488   0.1814
============================================================
R-squared            0.787655    Mean dependent var-3.840417
Adjusted R-squared   0.759343    S.D. dependent var 19.54705
S.E. of regression   9.589161    Akaike info criteri7.510156
Sum squared resid    1379.280    Schwarz criterion  7.658551
Log likelihood      -64.59140    F-statistic        27.81995
Durbin-Watson stat   2.511362    Prob(F-statistic)  0.000009
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```
============================================================
Dependent Variable: Y
Method: Least Squares
Date: 05/06/07   Time: 15:00
Sample: 1 18
Included observations: 18
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           0.405416   1.449845   0.279627   0.7833
        X3           4.020052   0.312887   12.84826   0.0000
============================================================
R-squared                        Mean dependent var-3.840417
Adjusted R-squared               S.D. dependent var 19.54705
S.E. of regression   5.989260    Akaike info criteri6.522252
Sum squared resid                Schwarz criterion  6.621182
Log likelihood      -56.70027    F-statistic        165.0777
Durbin-Watson stat   2.481735    Prob(F-statistic)  0.000000
============================================================
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============================================================
Dependent Variable: U
Method: Least Squares
Date: 05/06/07   Time: 15:02
Sample: 1 18
Included observations: 18
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C          -1.36E-16   1.449845  -9.36E-17   1.0000
        X3          -1.75E-16   0.312887  -5.60E-16   1.0000
============================================================
R-squared            0.000000    Mean dependent var 9.87E-17
Adjusted R-squared  -0.062500    S.D. dependent var 5.810436
S.E. of regression   5.989260    Akaike info criteri6.522252
Sum squared resid    573.9398    Schwarz criterion  6.621182
Log likelihood      -56.70027    F-statistic        3.17E-15
Durbin-Watson stat   2.481735    Prob(F-statistic)  1.000000
============================================================
```

```
============================================================
Dependent Variable: Y
Method: Least Squares
Date: 05/06/07   Time: 15:02
Sample: 1 18
Included observations: 18
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           0.839998   2.418470   0.347326   0.7329
        X1           4.264453   0.599350   7.115128   0.0000
============================================================
R-squared            0.759850    Mean dependent var-3.840417
Adjusted R-squared   0.744841    S.D. dependent var 19.54705
S.E. of regression   9.873859    Akaike info criteri7.522098
Sum squared resid    1559.889    Schwarz criterion  7.621028
Log likelihood      -65.69888    F-statistic        50.62505
Durbin-Watson stat   2.588269    Prob(F-statistic)  0.000002
============================================================


============================================================
Dependent Variable: U
Method: Least Squares
Date: 05/06/07   Time: 15:02
Sample: 1 18
Included observations: 18
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           1.86E-15   2.418470   7.68E-16   1.0000
        X1           1.71E-15   0.599350   2.86E-15   1.0000
============================================================
R-squared            0.000000    Mean dependent var-4.93E-17
Adjusted R-squared  -0.062500    S.D. dependent var 9.579050
S.E. of regression   9.873859    Akaike info criteri7.522098
Sum squared resid    1559.889    Schwarz criterion  7.621028
Log likelihood      -65.69888    F-statistic        2.33E-15
Durbin-Watson stat   2.588269    Prob(F-statistic)  1.000000
============================================================
```

Dependent Variable: U
Method: Least Squares
Date: 05/06/07 Time: 15:02
Sample: 1 18
Included observations: 18

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.199018	1.404188	-0.141732	0.8893
X1	-3.534387	0.703965	-5.020683	0.0002
X2	3.853749	3.099397	1.243387	0.2341
X3	3.168290	0.598357	5.294979	0.0001

R-squared	0.705519	Mean dependent var	-4.93E-17
Adjusted R-squared	0.642416	S.D. dependent var	9.579050
S.E. of regression	5.728111	Akaike info criteri	6.521779
Sum squared resid	459.3576	Schwarz criterion	6.719639
Log likelihood	-54.69601	F-statistic	11.18043
Durbin-Watson stat	2.317055	Prob(F-statistic)	0.000520