Econ 7331 Professor Chris Murray
Econometrics 1 McElhinney 211 D
Spring 2006

Final Exam

Write your answers on the blank paper that I have given you. Do not write your answers on this exam. **You must explain your answers.** If you are confused about a question or you think it is unclear, please ask for clarification before answering**. When testing a hypothesis, make sure to write down the null and alternative, the critical value(s), the test statistic, and your decision (reject or fail to reject).** Use tests with 5% size. Unless otherwise specified, use 2-sided alternative hypotheses.

1. Suppose you have data, which are *iid* exponential. That is

$$f(z_i;\theta) = \frac{1}{\theta} e^{-z_i/\theta}$$

where $0 \le z_i < \infty$, and $\theta > 0$. Also, $E(z_i) = \theta$ and $\text{var}(z_i) = \theta^2$.

a. Derive the log likelihood function. **(5 points)**

b. Derive the maximum likelihood estimator of θ, $\hat{\theta}_{MLE}$. Is $\hat{\theta}_{MLE}$ unbiased? **(5 points)**

c. Derive the Cramér-Rao lower bound for unbiased estimators of θ. Does $\hat{\theta}_{MLE}$ achieve the Cramér-Rao lower bound? **(5 points)**

d. Suppose $N = 100$ and $\sum_{i=1}^{N} z_i = 60$. Perform the Wald test that $\theta = 0.5$. **(10 points)**

Let $\gamma = g(\theta) = \frac{1}{\theta^2}$

e. Derive the asymptotic variance of $\hat{\gamma}$. **(5 points)**

f. Find a consistent estimator of the asymptotic variance of $\hat{\gamma}$. **(5 points)**

g. Test the null hypothesis that $\gamma = 4$. **(10 points)**

2. Consider the classical linear regression model $y = X\beta + u$. Assume that the usual regression model assumptions hold, except that the error terms are distributed as follows: $u \sim (0, \sigma^2\Sigma)$, where Σ is an n x n symmetric positive definite matrix.

a. Prove that $\hat{\beta}_{OLS} = (X'X)^{-1}X'y$ is unbiased. **(7 points)**

b. Derive the variance-covariance matrix of $\hat{\beta}_{OLS}$. **(7 points)**

c. Discuss the optimality, or lack thereof, of $\hat{\beta}_{OLS}$ when $\Sigma \neq I$. **(7 points)**

d. Discuss the consequences of doing OLS based inference failing to take into account that $\Sigma \neq I$. **(7 points)**

3. Consider the regression model

$$q = Z\gamma + v,$$

where the standard assumptions hold. q is the dependent variable, Z is the matrix of independent variables, γ is the vector of unknown regression parameters, and v is the error term.

a. What is the residual maker for the matrix Z? Call this matrix M. **(5 points)**

b. What is the value of MZ? Give the intuition behind this result. **(5 points)**

c. What is the matrix which projects onto the space spanned by the regressors? Call this matrix P. **(5 points)**

d. What is the value of PZ? Give the intuition behind this result. **(5 points)**

e. What is the OLS decomposition of q into its explained and unexplained components? In what specific sense is this decomposition an orthogonal one? **(5 points)**

Suppose that we partition the matrix of regressors as follows:

$$q = Z_1\gamma_1 + Z_2\gamma_2 + v$$

f. What is the OLS estimator of γ_1? **(10 points)**

4. Consider the following regression model:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i,$$

where $u_i \sim iid(0, \sigma^2)$.

Use the regression output on the following pages to test the hypotheses below. The dependent variable "u" is the residual from the most recent regression with "y" as the dependent variable.

a. $H_0 : \beta_2 = 1$
$H_1 : \beta_2 \neq 1$ **(5 points)**

b. $H_0 : \beta_1 = 0, \beta_3 = 0$
$H_1 : \beta_1 \neq 0, \beta_3 \neq 0$ **(10 points)**

c. $H_0 : \beta_3 = 0$
$H_1 : \beta_3 \neq 0$ **(10 points)**

For the remainder of this question, assume that $u_i \sim iidN(0, \sigma^2)$.

d. $H_0 : \beta_2 = 0, \beta_3 = 0$
$H_1 : \beta_2 \neq 0, \beta_3 \neq 0$ **(10 points)**

e. Use a confidence interval to test the null that $\beta_0 = 3$. **(5 points)**

Regression Output for Question 4

```
============================================================
Dependent Variable: Y
Method: Least Squares
Date: 05/01/06   Time: 16:32
Sample: 1 20
Included observations: 20
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           3.478189   0.942889   3.688862   0.0020
        X1           1.800640   0.488314   3.687463   0.0020
        X2           2.417693   2.028733   1.191726   0.2507
        X3           0.121182   0.399914   0.303021   0.7658
============================================================
R-squared            0.833641    Mean dependent var 1.141134
Adjusted R-squared   0.802449    S.D. dependent var 9.041927
S.E. of regression   4.018837    Akaike info criteri5.796718
Sum squared resid    258.4168    Schwarz criterion  5.995865
Log likelihood      -53.96718    F-statistic        26.72594
Durbin-Watson stat   2.334340    Prob(F-statistic)  0.000002
============================================================


               Coefficient Covariance Matrix
============================================================
                  C           X1          X2          X3
============================================================
     C        0.889040    0.081772   -0.158388    0.000495
     X1       0.081772    0.238450   -0.318135   -0.149840
     X2      -0.158388   -0.318135    4.115756   -0.084983
     X3       0.000495   -0.149840   -0.084983    0.159931
============================================================



============================================================
Dependent Variable: Y
Method: Least Squares
Date: 05/01/06   Time: 16:50
Sample: 1 20
Included observations: 20
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           3.574232   0.929266   3.846293   0.0012
        X1           2.156711   0.240626   8.962928   0.0000
============================================================
R-squared            0.816951    Mean dependent var 1.141134
Adjusted R-squared   0.806781    S.D. dependent var 9.041927
S.E. of regression   3.974530    Akaike info criteri5.692330
Sum squared resid    284.3440    Schwarz criterion  5.791903
Log likelihood                   F-statistic        80.33408
Durbin-Watson stat   2.602264    Prob(F-statistic)  0.000000
============================================================
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============================================================
Dependent Variable: U
Method: Least Squares
Date: 05/01/06   Time: 16:50
Sample: 1 20
Included observations: 20
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C          -0.096043   0.942889  -0.101860   0.9201
        X1          -0.356071   0.488314  -0.729185   0.4764
        X2           2.417693   2.028733   1.191726   0.2507
        X3           0.121182   0.399914   0.303021   0.7658
============================================================
R-squared                        Mean dependent var 5.61E-16
Adjusted R-squared               S.D. dependent var 3.868523
S.E. of regression   4.018837    Akaike info criteri5.796718
Sum squared resid    258.4168    Schwarz criterion  5.995865
Log likelihood      -53.96718    F-statistic        0.535099
Durbin-Watson stat   2.334340    Prob(F-statistic)  0.664845
============================================================
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```
============================================================
Dependent Variable: Y
Method: Least Squares
Date: 05/01/06   Time: 16:51
Sample: 1 20
Included observations: 20
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           1.872611   1.560582   1.199944   0.2457
        X2           10.24604   2.701390   3.792877   0.0013
============================================================
R-squared                        Mean dependent var 1.141134
Adjusted R-squared               S.D. dependent var 9.041927
S.E. of regression               Akaike info criteri6.802977
Sum squared resid                Schwarz criterion  6.902550
Log likelihood      -66.02977    F-statistic        14.38592
Durbin-Watson stat   1.377635    Prob(F-statistic)  0.001332
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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 05/01/06   Time: 16:51
Sample: 1 20
Included observations: 20
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           3.571230   0.951194   3.754471   0.0016
        X1           1.987520   0.468130   4.245657   0.0005
        X3           0.171103   0.402599   0.424996   0.6762
============================================================
R-squared            0.818875    Mean dependent var 1.141134
Adjusted R-squared   0.797566    S.D. dependent var 9.041927
S.E. of regression   4.068203    Akaike info criteri5.781761
Sum squared resid    281.3546    Schwarz criterion  5.931121
Log likelihood      -54.81761    F-statistic        38.42891
Durbin-Watson stat   2.575777    Prob(F-statistic)  0.000000
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============================================================
Dependent Variable: U
Method: Least Squares
Date: 05/01/06   Time: 16:51
Sample: 1 20
Included observations: 20
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C          -0.093041   0.942889  -0.098676   0.9226
        X1          -0.186880   0.488314  -0.382705   0.7070
        X2           2.417693   2.028733   1.191726   0.2507
        X3          -0.049921   0.399914  -0.124830   0.9022
============================================================
R-squared            0.081527    Mean dependent var 2.05E-16
Adjusted R-squared  -0.090687    S.D. dependent var 3.848134
S.E. of regression   4.018837    Akaike info criteri5.796718
Sum squared resid    258.4168    Schwarz criterion  5.995865
Log likelihood      -53.96718    F-statistic        0.473404
Durbin-Watson stat   2.334340    Prob(F-statistic)  0.705107
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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 05/01/06   Time: 16:51
Sample: 1 20
Included observations: 20
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           3.477814   0.917357   3.791123   0.0015
        X1           1.914176   0.304674   6.282693   0.0000
        X2           2.482086   1.962941   1.264473   0.2231
============================================================
R-squared                        Mean dependent var 1.141134
Adjusted R-squared               S.D. dependent var 9.041927
S.E. of regression   3.910016    Akaike info criteri5.702441
Sum squared resid                Schwarz criterion  5.851801
Log likelihood                   F-statistic        42.30292
Durbin-Watson stat   2.355179    Prob(F-statistic)  0.000000
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============================================================
Dependent Variable: U
Method: Least Squares
Date: 05/01/06   Time: 16:52
Sample: 1 20
Included observations: 20
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           8.52E-16   0.917357   9.29E-16   1.0000
        X1           8.01E-16   0.304674   2.63E-15   1.0000
        X2          -3.29E-15   1.962941  -1.68E-15   1.0000
============================================================
R-squared            0.000000    Mean dependent var 1.70E-16
Adjusted R-squared  -0.117647    S.D. dependent var 3.698504
S.E. of regression   3.910016    Akaike info criteri5.702441
Sum squared resid    259.8998    Schwarz criterion  5.851801
Log likelihood      -54.02441    F-statistic        1.86E-15
Durbin-Watson stat   2.355179    Prob(F-statistic)  1.000000
============================================================
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```
============================================================
Dependent Variable: U
Method: Least Squares
Date: 05/01/06   Time: 16:52
Sample: 1 20
Included observations: 20
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           0.000375   0.942889   0.000398   0.9997
        X1          -0.113536   0.488314  -0.232506   0.8191
        X2          -0.064393   2.028733  -0.031740   0.9751
        X3           0.121182   0.399914   0.303021   0.7658
============================================================
R-squared            0.005706    Mean dependent var 1.70E-16
Adjusted R-squared  -0.180724    S.D. dependent var 3.698504
S.E. of regression   4.018837    Akaike info criteri5.796718
Sum squared resid    258.4168    Schwarz criterion  5.995865
Log likelihood      -53.96718    F-statistic        0.030607
Durbin-Watson stat   2.334340    Prob(F-statistic)  0.992507
============================================================
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