Econ 7331 Professor Chris Murray
Econometrics 1 McElhinney 211 D
Spring 2005

Final Exam

Please write your answers on your own paper. **You must explain your answers.** If you are confused about a question or you think it is unclear, please ask for clarification before answering**. When testing a hypothesis, make sure to write down the null and alternative, the critical value(s), the test statistic, and your decision (reject or fail to reject).** Use tests with 5% size. Unless otherwise specified, use 2-sided alternative hypotheses.

1. Suppose you have data, which are *iid* Bernoulli. That is

$$f(x_i;\theta) = \theta^{x_i}(1-\theta)^{1-x_i}$$

where $x_i = 0,1$. Also, $0 \le \theta \le 1$, $E(x_i) = \theta$ and $\text{var}(x_i) = \theta(1-\theta)$.

a. Derive the likelihood function for a sample of data, i = 1, 2, …, N. **(3 points)**

b. Derive the log likelihood function. **(2 points)**

c. Derive the maximum likelihood estimator of θ, $\hat{\theta}_{MLE}$. Is $\hat{\theta}_{MLE}$ unbiased? **(5 points)**

d. Derive the Cramér-Rao lower bound for unbiased estimators of θ. Does $\hat{\theta}_{MLE}$ achieve the Cramér-Rao lower bound? **(8 points)**

Let N = 30 and $\sum_{i=1}^{N} x_i = 18$.

e. Perform a Wald test of the null that $\theta = 0.5$ vs. the 2-sided alternative. **(10 points)**

f. Perform a Lagrange Multiplier test of the null that $\theta = 0.5$ vs. the 2-sided alternative. **(12 points)**

2. Consider the following linear regression model:

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

a. Discuss the statistical properties of $\hat{\beta}_{1,OLS}$ if the error terms are heteroskedastic. **(5 points)**

b. Discuss the consequences of using OLS standard errors (which assume homoskedasticity) when the error terms are heteroskedastic. **(5 points)**

c. Using the following regression output, perform White's test for heteroskedasticity. **(5 points)**

d. Based on the outcome of White's test, test the following null hypothesis: **(5 points)**

$$H_0 : \beta_1 = 1$$
$$H_1 : \beta_1 \neq 1$$

e. Discuss the optimality, or lack thereof, of $\hat{\beta}_{1,OLS}$ when White's heteroskedasticity-consistent standard errors are used. **(5 points)**

Regression Output for Question 2

Dependent Variable: Y
Method: Least Squares
Date: 04/28/05 Time: 14:18
Sample: 1 150
Included observations: 150

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.180345	0.435139	0.414453	0.6791
X	1.027454	0.192296	5.343077	0.0000

R-squared	0.161703	Mean dependent var	0.086454
Adjusted R-squared	0.156039	S.D. dependent var	5.796398
S.E. of regression	5.324997	Akaike info criteri	6.195945
Sum squared resid	4196.628	Schwarz criterion	6.236087
Log likelihood	-462.6959	F-statistic	28.54847
Durbin-Watson stat	2.022659	Prob(F-statistic)	0.000000

White Heteroskedasticity Test:

F-statistic	5.521783	Probability	0.004872
Obs*R-squared	10.48151	Probability	0.005296

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 04/28/05 Time: 14:18
Sample: 1 150
Included observations: 150

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	18.31229	5.784292	3.165865	0.0019
X	2.422824	2.079284	1.165220	0.2458
X^2	1.930787	0.666149	2.898431	0.0043

R-squared	0.069877	Mean dependent var	27.97752
Adjusted R-squared	0.057222	S.D. dependent var	58.60855
S.E. of regression	56.90700	Akaike info criteri	10.94051
Sum squared resid	476045.8	Schwarz criterion	11.00072
Log likelihood	-817.5383	F-statistic	5.521783
Durbin-Watson stat	1.466213	Prob(F-statistic)	0.004872

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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 04/28/05   Time: 14:18
Sample: 1 150
Included observations: 150
White Heteroskedasticity-Consistent Standard Errors & Covariance
============================================================
    Variable    Coefficient   Std. Error  t-Statistic  Prob.
============================================================
         C          0.180345   0.440112   0.409770   0.6826
         X          1.027454   0.253318   4.055983   0.0001
============================================================
R-squared             0.161703    Mean dependent var 0.086454
Adjusted R-squared    0.156039    S.D. dependent var 5.796398
S.E. of regression    5.324997    Akaike info criteri6.195945
Sum squared resid     4196.628    Schwarz criterion  6.236087
Log likelihood       -462.6959    F-statistic        28.54847
Durbin-Watson stat    2.022659    Prob(F-statistic)  0.000000
============================================================
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3. Consider the following linear regression model

$$y = X\beta + u$$

where we partition the matrix of explanatory variables as follows:

$$y = X_1\beta_1 + X_2\beta_2 + u\,.$$

Suppose that interest centers on β_1 only.

State and prove the Frisch-Waugh theorem. **(25 points)**

4. Consider the following regression model with Gaussian errors:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i,$$

and $u_i \sim iidN(0, \sigma^2)$.

a. Test the following hypothesis: **(5 points)**

$$H_0 : \beta_2 = 2$$
$$H_1 : \beta_2 \neq 2$$

b. Test the following hypotheses: **(5 points)**

$$H_0 : \beta_1 = 0, \beta_2 = 0$$
$$H_1 : \beta_1 \neq 0, \beta_2 \neq 0$$

Assume now that the errors are non Gaussian.

c. Test the following hypothesis: **(5 points)**

$$H_0 : \beta_1 + \beta_3 = 4$$
$$H_1 : \beta_1 + \beta_3 \neq 4$$

d. Test the following hypotheses: **(5 points)**

$$H_0 : \beta_1 = 0, \beta_3 = 0$$
$$H_1 : \beta_1 \neq 0, \beta_3 \neq 0$$

e. Construct a 95% confidence interval for $\hat{\beta}_3$. Use this confidence interval to test the hypothesis that the intercept is $\beta_3 = 3$ vs. the 2-sided alternative: **(5 points)**

Regression Output for Question 4

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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 04/28/05   Time: 14:40
Sample: 1 10
Included observations: 10
============================================================
    Variable    Coefficient   Std. Error  t-Statistic  Prob.
============================================================
         C           0.092487   0.330572   0.279777   0.7890
        X1           1.035834   0.153804   6.734757   0.0005
        X2           1.693281   0.705156   2.401287   0.0532
        X3           3.034119   0.130914   23.17635   0.0000
============================================================
R-squared            0.998117    Mean dependent var-4.137519
Adjusted R-squared   0.997176    S.D. dependent var 18.95786
S.E. of regression   1.007478    Akaike info criteri3.141952
Sum squared resid    6.090072    Schwarz criterion  3.262986
Log likelihood      -11.70976    F-statistic        1060.255
Durbin-Watson stat   1.509289    Prob(F-statistic)  0.000000
============================================================

               Coefficient Covariance Matrix
============================================================
                C           X1           X2           X3
============================================================
     C       0.109278    0.008139    0.006293   -0.001494
     X1      0.008139    0.023656   -0.018575   -0.016110
     X2      0.006293   -0.018575    0.497245   -0.013007
     X3     -0.001494   -0.016110   -0.013007    0.017139
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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 04/28/05   Time: 14:41
Sample: 1 10
Included observations: 10
============================================================
    Variable    Coefficient   Std. Error  t-Statistic  Prob.
============================================================
         C           0.356961   2.910154   0.122661   0.9058
        X1           3.887856   0.812718   4.783769   0.0020
        X2           3.996020   6.149486   0.649814   0.5365
============================================================
R-squared            0.829563    Mean dependent var-4.137519
Adjusted R-squared   0.780867    S.D. dependent var 18.95786
S.E. of regression   8.874498    Akaike info criteri7.447566
Sum squared resid    551.2970    Schwarz criterion  7.538341
Log likelihood      -34.23783    F-statistic        17.03543
Durbin-Watson stat   2.938817    Prob(F-statistic)  0.002044
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==============================================================
Dependent Variable: Y
Method: Least Squares
Date: 04/28/05   Time: 14:41
Sample: 1 10
Included observations: 10
==============================================================
    Variable    Coefficient   Std. Error  t-Statistic  Prob.
==============================================================
         C          0.071055   0.428427   0.165852   0.8730
        X1          1.099087   0.196459   5.594482   0.0008
        X3          3.078413   0.168036   18.31999   0.0000
==============================================================
R-squared             0.996308    Mean dependent var-4.137519
Adjusted R-squared    0.995253    S.D. dependent var 18.95786
S.E. of regression    1.306184    Akaike info criteri3.615422
Sum squared resid     11.94281    Schwarz criterion  3.706197
Log likelihood       -15.07711    F-statistic        944.4442
Durbin-Watson stat    1.872207    Prob(F-statistic)  0.000000
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==============================================================
Dependent Variable: Y
Method: Least Squares
Date: 04/28/05   Time: 14:41
Sample: 1 10
Included observations: 10
==============================================================
    Variable    Coefficient   Std. Error  t-Statistic  Prob.
==============================================================
         C          0.314657   2.802401   0.112281   0.9134
        X1          4.140400   0.687514   6.022273   0.0003
==============================================================
R-squared             0.819282    Mean dependent var-4.137519
Adjusted R-squared    0.796692    S.D. dependent var 18.95786
S.E. of regression    8.548046    Akaike info criteri7.306139
Sum squared resid     584.5527    Schwarz criterion  7.366656
Log likelihood       -34.53069    F-statistic        36.26778
Durbin-Watson stat    2.872118    Prob(F-statistic)  0.000315
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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 04/29/05   Time: 09:01
Sample: 1 10
Included observations: 10
============================================================
    Variable    Coefficient   Std. Error  t-Statistic  Prob.
============================================================
         C          -2.718727   5.485638  -0.495608   0.6335
        X2           18.06355   10.43843   1.730484   0.1218
============================================================
R-squared              0.272368    Mean dependent var-4.137519
Adjusted R-squared     0.181415    S.D. dependent var 18.95786
S.E. of regression     17.15226    Akaike info criteri8.698994
Sum squared resid      2353.601    Schwarz criterion  8.759511
Log likelihood                     F-statistic        2.994576
Durbin-Watson stat     1.968410    Prob(F-statistic)  0.121792
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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 04/28/05   Time: 14:43
Sample: 1 10
Included observations: 10
============================================================
    Variable    Coefficient   Std. Error  t-Statistic  Prob.
============================================================
         C          -0.329758   0.924192  -0.356807   0.7305
        X3           3.872788   0.196604   19.69838   0.0000
============================================================
R-squared                          Mean dependent var-4.137519
Adjusted R-squared                 S.D. dependent var 18.95786
S.E. of regression     2.857911    Akaike info criteri5.114915
Sum squared resid      65.34122    Schwarz criterion  5.175432
Log likelihood        -23.57458    F-statistic        388.0264
Durbin-Watson stat     1.803396    Prob(F-statistic)  0.000000
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