

U.S. Congressional Vote Empirics: A Discrete Choice Model of Voting

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Abstract

This paper uses United States congressional district level data to identify how incumbency, candidate campaign expenditures and district voter registration characteristics influence final vote shares. We estimate the factor impacts using a representative agent-based discrete choice model and an estimation method similar to Berry, Levinsohn, and Pakes (1995). District-level partisan voter registration statistics provide the observable distribution of voter heterogeneity, therefore, it is not necessary to estimate the voter heterogeneity distribution parameters. This allows for the parameters to be estimated with precision even though there are a relatively small number of observations. Moreover, the structural discrete demand model with voter heterogeneity provides a general methodology of factor impact estimates by which we can formally test the modeling restrictions found in Levitt (1995) and Gerber (1998).

1 Introduction

While campaign expenditures is a common topic of discussion in the media during every election cycle, the issue of redistricting congressional districts is becoming more prevalent. In previous decades, state governments only redrew the lines of their congressional districts after each US Census was completed. This was necessary to ensure that each congressional district contained a similar number of people. However, state governments are starting to break that tradition. During this decade, the legislatures of Texas and Georgia's state legislature switched from a Democratic majority to a Republican majority. The new Republican majority then redrew the boundaries of the congressional districts, resulting in voter distributions of that favored the Republican party. Texas went from having 17 Republican and 15 Democratic US House Representatives before the redistricting to 21 Republican and 11 Democratic Representatives after. This shows that by controlling the distribution of registered voters, redistricting can have real political power and a political party can exert greater Congressional influence in the determination of public policy.

The purpose of this paper is to estimate the effects of expenditures, incumbency, and makeup of registered voters in each district on observed aggregated district vote results. In the previous estimates of vote shares, the distribution of voters within the congressional district has not been incorporated in the analysis. Most have centered on the advantages from increasing expenditures and being an incumbent candidate. This paper will use the fact that voters in certain states have to register for a certain political party to vote in primary elections. This not only allows the estimates to control for the district partisanship, it also allows for estimates of expenditure and incumbency effects on the heterogeneous population of registered voters.

There have been many studies in the economics and political science literature that seek to explain the relationship between earned vote shares and candidate characteristics¹.

¹See Gerber (2004) for a thorough summary of the reduced form empirical literature.

Gerber (1998) and Levitt (1994) both estimate electoral factor impacts using reduced-form models. They differ in their techniques to control for the endogeneity that final vote shares and candidate expenditures may be co-caused by a factor such as candidate quality. An eloquent or charismatic candidate may garner more supporters, more votes, more financial support, and have larger expenditures. Gerber employs 2 Stage Least Squares and tries several instruments. Levitt introduces the unobserved qualitative regressor, but then uses repeat electoral races to difference it out and obtain consistent estimates.

Berry, Levinsohn, and Pakes (1995) (hereafter BLP) introduce a structural discrete choice demand model which has been employed across many industries to estimate the impacts of product characteristics using observed product market share. In this paper, we apply their methodology to explain the impact of candidate characteristics on observed candidate vote share received. We modify the discrete product-demand estimation methodology from BLP for observed aggregate vote share data. In our model, the market is a congressional district, the products are the candidates seeking elections, the characteristics are incumbency status and expenditures, the market share is the vote share and the consumer heterogeneity random-coefficient distribution is the partisan voter registration distribution.

This paper contributes to the campaign expenditure literature in two ways. First, the discrete distribution of district partisan voter registration statistics are introduced into the BLP model to account for voter heterogeneity. Rekkas (2007) is the first and only paper until this one to apply the BLP methodology to election data. She analyzes the 1997 Canadian Parliamentary Election. She uses the same methodology used in BLP. The main differences between our work is that she did not have voter registration statistics and thus had to estimate parameters of the random-coefficient heterogeneity distribution, and we are analyzing many years of US House of Representative elections in the United States. The major drawback of Rekkas (2007) is that the Canadian elections are generally multi-party, which requires estimating more parameters in an already parameter-heavy framework. This

leads to very imprecise estimates of the parameters in the full model. Fortunately, our data have allowed us to achieve a high degree of accuracy because we do not need to estimate the population distributions. In fact, the observed voter registration data provides an additional source of variation that improves the precision of our estimates.

The second major contribution of this paper is that the earlier reduced form studies of Levitt (1994) and Gerber (1998) are special cases of our structural model. Levitt (1994) introduced the unobserved candidate regressor and our model also includes this regressor. However, instead of differencing the equations to remove this regressor, our method explicitly solves for this unobserved regressor. We then use instruments similar to Gerber (1998) to deal with the endogeneity between the unobserved regressor and the candidate's expenditure. The main difference between our work and these two studies is that we estimate a structural model instead of positing a reduced form. In fact, the Levitt and Gerber models are specific cases of our model. Both studies implicitly imply that there is no heterogeneity in the voter population. Therefore, we empirically test the set of parameter restrictions on our model that yield the Levitt (1994) and Gerber (1998) regressions. We show that the voters who are registered to different parties respond differently to the candidate characteristics.

The organization for the remainder of this paper is as follows. Section 2 presents the discrete choice demand model of voting and provides the general estimation equation. It further introduces the voter registration distribution as the random-coefficient distribution, and briefly explains how to estimate the parameters using observed vote shares. Section 3 explains the data used for this analysis. Section 4 provides and interprets the empirical results. Then we compare our results with the previous literature and test the equating restrictions. Section 5 concludes.

2 Discrete Choice Model Specification

In this section, as is standard in the discrete choice literature, we derive the probability of voting for each of the candidates. In our model, there are three different types of representative agents in every congressional district that each have a random taste shock for both of the candidates. As will be shown, the mean utility of each candidate and an assumed distribution for the random tastes will completely determine the probability that each type of agent votes for each candidate. Then, the predicted market share function for each candidate is the average of the three probabilities weighted by the proportion of each type of agent in the market. Our estimation of the parameters of the model is based on the fact that there are covariates that are orthogonal to the model error that occurs because candidate quality is unobserved.

The main difference between our methodology and the previous literature is that we are able to observe the proportion of each type of agent in each market. This provides us two main advantages. First, there is no need to estimate the distribution parameters so the estimation is much more precise. Second and more importantly, we are able estimate different parameters for the different agents. Instead of having a mean and variance of the parameter over the distribution, we estimate different mean values for each type of agent. This provides much more detailed information of how voter utility responds to the candidate's characteristics.

2.1 Utility of a Representative Agent

We assume that there are three representative utility-maximizing agents in each district. One type is a voter who is registered for the Democratic Party, the second type is a voter registered for the Republican Party, and the third type is a voter who is not registered for either of those two parties. Each type of agent is facing a discrete voting decision: whether to vote for the candidate from the Democratic Party, the Republican Party, or not vote for

one of the two candidates from the dominant parties. The third option can be considered the outside option. It includes the choice of voting for a third party candidate. The outside option is defined this way because, for the past decade, every representative to the US House has been a member of the Democratic or Republican party². The voting decision is modeled as follows:

$$\max_{j \in \{0,1,N\}} U_{nmj} = V_n(p_{mj}, x_{mj}) + e_{nmj} \quad (1)$$

From this point on, the Democratic Party is by the subscript 0, the Republican Party by the subscript 1, and registered voters who did not pick one of those parties is represented by the subscript N. For this decision problem, the j subscript denotes the different voting options: Democratic, Republican, and Neither. The m subscript denotes the market which is defined as a congressional district and the n subscript denotes the three different types of agents. For example, U_{0m1} is the utility that a registered Democrat in congressional district m receives from voting for the Republican candidate. It is important to note that this utility function is defined as the utility of voting for a specific candidate and not the utility from having that candidate be your representative³.

The p_{mj} variable is the campaign expenditures by candidates j in market m , and x_{mj} is a vector of observable candidate and market characteristics. We define x_{mj} as the vector of control variables (Incumbent, Senate Race, Presidential Race, Democratic Party Dummy Variable, State Dummy Variables, and a Constant)⁴. Therefore, $V_n(\cdot)$ is the indirect utility from all of the observable characteristics and e_{nmj} is the unobserved utility that equates $V_n(\cdot)$ and the actual utility of each individual. We assume that $V_n(\cdot)$ is linear in its arguments.

There are two sources of unobserved utility. The first is the utility from the candidate

²While Rekkas (2007) includes all the minor parties when analyzing Canadian elections, Lee (2008) makes the same outside option assumption that we do since the United States government is completely dominated by the two major parties.

³This distinction is discussed more in the extensions.

characteristics that are observed by the representative agent but unobserved by the econometrician, denoted by ξ_{mj} . These unobserved candidate characteristics are different for each candidate in each market but the same across the different agents within a market. Hence, there is no “n” subscript since the utility does not vary by agent. The second source of unobserved utility is the idiosyncratic utility shock, denoted by ϵ_{nmj} . This shock represents the unobserved “taste” for each candidate that varies by agent. Therefore, $e_{nj} = \xi_{mj} + \epsilon_{nmj}$ and we rewrite equation 1 as:

$$U_{nmj} = \alpha_n \ln(p_{mj}) + \beta_n x_{mj} + (\xi_{mj} + \epsilon_{nmj}) \quad (2)$$

2.2 Probability of Voting

While U_{nmj} is not observable, using the fact that each agent is a utility maximizer and assuming a distribution on ϵ_{nmj} allows us to derive the probability that each type of agent will vote for each of the candidates. First, define the mean utility level for each candidate in each market as:

$$\delta_{nmj} \equiv \alpha_n \ln(p_{mj}) + \beta_n x_{mj} + \xi_{mj} \quad (3)$$

$$\therefore \text{Equation 2 is simplified to } U_{nmj} = \delta_{nmj} + \epsilon_{nmj} \quad (4)$$

Simplifying the utility this way shows that the δ_{nmj} and the individual shock completely determine the voting choice for each individual, thereby completely determining the probability that the representative agent votes for each candidate. However, note that in each market m , ξ_{mj} is the same for each n . This restriction is necessary to estimate the parameters since we do not observe individual voting choices of the different types of agent. From here on we will drop the m subscript to make the equations more concise.

Since the outside option does not have any candidate characteristics and a discrete choice model can only identify relative utility levels, we redefine ϵ_{nj} as utility shock difference

with the outside choice shock and normalize $\delta_{nN} \equiv 0$. Therefore, within each market, the probability of the representative agent of each type voting for each candidate can be found as:

$$P_{nj} = \int_{B_{nj}} dP(\epsilon_{nj}, \epsilon_{nk}) \text{ where } B_{nj} = \{\epsilon_{nj}, \epsilon_{nk} | U_{nj} > U_{nk} \forall j \neq k\} \quad (5)$$

$$\therefore B_{nj} = \{(\epsilon_{nj}, \epsilon_{nk}) | \delta_{nj} + \epsilon_{nj} > 0; \delta_{nj} + \epsilon_{nj} > \delta_{nk} + \epsilon_{nk}\} \quad (6)$$

We assume that all ϵ_{nj} are independently distributed type 1 extreme value, $P(\epsilon_{nj}) = \exp(-\exp(-\epsilon_{nj}))$. As in standard BLP, the assumptions of the model lead to the well known result that the predicted probability of each type of agent voting for candidate j has the logit form⁴.

$$P_{nj} = \frac{e^{\delta_{nj}}}{1 + \sum_{j \in \{0,1\}} e^{\delta_{nj}}} \quad (7)$$

2.3 The Discrete Distribution of Agents and Random Coefficients

As indicated above, we are assuming there is a separate utility functions for the three different type of agents. The functional form of utility is the same for each agent, but the coefficients on the candidate characteristics can differ by agent. Therefore, this is a random-coefficients utility model similar to BLP. BLP introduces random coefficients into the model under the argument that different product characteristics can influence heterogeneous consumers differently. This has the dual virtues of being more realistic and alleviating the substitution pattern restrictions imposed by a standard logit model. The additional parameters provide explicit information on how candidate characteristics affect the different registration groups. For example, α_{10} measures the utility effect (and hence vote share effect) for a registered Democrat from an increase in expenditure by the Republican candidate.

⁴See Train (2003) pg. 78 for the algebra

Our model relates significantly to the model in Berry, Carnall and Spiller (2006) as well as to BLP. This is do to the assumption of a discrete distribution of heterogeneous agents. BLP assume the coefficients are distributed normally and estimate the distribution parameters for each coefficient. Berry, et al (2006) assume that the there are two discrete type of agents and estimate the percentage of each type of individual. Unlike Berry, et al (2006), we observe the percentage of people in each group in each district and therefore do not have to estimate the percentages for each type of agent or argue that they are identified. In each market m , let:

$$\begin{aligned}\mu_{m0} &\equiv \frac{\# \text{ of registered Democrats in district}}{\text{total } \# \text{ of registered voters in market } m} \\ \mu_{m1} &\equiv \frac{\# \text{ of registered Republicans in district}}{\text{total } \# \text{ of registered voters in market } m} \\ \mu_{mN} &\equiv 1 - \mu_{m0} - \mu_{m1}\end{aligned}$$

2.4 Predicted Market Share Function

The probability that an agent of type n will vote for j and the proportion of each type of agent in each market are shown in the two previous sections. Combining them, the predicted vote share function for each candidate is the sum over the probability of each type of agent voting for candidate j weighted by the proportion of each type of voter.

$$\hat{s}_j(\alpha, \beta, \xi; \mu) = \sum_{n \in \{0,1,N\}} \mu_n * \frac{e^{\delta_{nj}}}{1 + \sum_{j \in \{0,1\}} e^{\delta_{nj}}} \quad (8)$$

2.5 Elasticities of Expenditure

The advantage of having the observed heterogeneity in the model is highlighted by the own and cross elasticities of expenditure. These explicitly elasticities show the how a percentage increase in expenditure will change the percentage of each type of agent that votes for him.

Define η_{jj}^* as the own expenditure elasticity. This is the percentage change in vote share for candidate j if he increases his expenditure by 1%. In our model:

$$\begin{aligned}\eta_{jj}^* &= \frac{\partial \hat{s}_j^*(\alpha, \beta, \xi; \mu)}{\partial p_j} \frac{p_j}{\hat{s}_j^*(\alpha, \beta, \xi; \mu)} \\ &= \frac{\sum_{n \in \{0,1,N\}} \alpha_n * \mu_n * \frac{e^{\delta_{nj}}}{1 + \sum_{j \in \{0,1\}} e^{\delta_{nj}}} \left[1 - \frac{e^{\delta_{nj}}}{1 + \sum_{j \in \{0,1\}} e^{\delta_{nj}}} \right]}{\hat{s}_j^*(\alpha, \beta, \xi; \mu)}\end{aligned}\quad (9)$$

The numerator of equation (9) shows that the overall change in vote share is the sum of the change in vote share from each type of agent. Therefore, our model provides a closed form answer for the change in voting behavior for each type of agent.

Similarly, define η_{jk}^* as the cross expenditure elasticity. This is the percentage change in vote share for candidate j from a 1% increase in expenditure by candidate k .

$$\begin{aligned}\eta_{jk}^* &= \frac{\partial \hat{s}_j^*(\alpha, \beta, \xi; \mu)}{\partial p_k} \frac{p_k}{\hat{s}_j^*(\alpha, \beta, \xi; \mu)} \\ &= \frac{- \sum_{n \in \{0,1,N\}} \alpha_n * \mu_n * \frac{e^{\delta_{nj}}}{1 + \sum_{j \in \{0,1\}} e^{\delta_{nj}}} \frac{e^{\delta_{nk}}}{1 + \sum_{j \in \{0,1\}} e^{\delta_{nj}}}}{\hat{s}_j^*(\alpha, \beta, \xi; \mu)}\end{aligned}\quad (10)$$

2.6 Endogeneity of Unobserved Candidate Characteristics

ξ_{mj} is defined as all the characteristics of the candidate that are unobserved by the econometrician but which affect the mean utility level of the candidate. Some of these characteristics are charisma, public speaking ability, physical appearance and performance in office for incumbents. These characteristics not only affect the number of votes a candidate receives, but they potentially affect the amount of contributions a candidate raises. Furthermore, a candidate with greater contributions can expend more on the congressional race. Without controlling for any other factors other than incumbency, this causes the unobserved

characteristics to be endogenous to the candidate's expenditure.⁵

However, our model controls the distribution of partisan voter in each district. Controlling for district partisan registration statistics in Congressional races may explain any indirect causation that biases the previous expenditure impact estimates. The voter registration statistics in each district are continuously changing. In fact, a lot of the candidate expenditure during each election is focused on registering people to vote and concurrently convincing people to register for the candidates party. If the unobserved candidate characteristics induce a person to register for that candidates party in a similar way that they induce a person to vote for the candidate, the registration statistics could capture some of the correlation between expenditures and the unobserved candidate characteristics. Therefore, controlling for the voter distribution in each district provides additional controls that may limit the endogeneity of expenditures.⁶Unfortunately, we are unable to statically test this claim using the standard Hausman test due to computational limitations. The estimation methods are highly nonlinear and take many hours to run so it would take many months to construct the Hausman statistic. In light of this fact, we will estimate the model ignoring endogeneity and also estimate it controlling for the presence of endogeneity.

2.7 Estimation

2.7.1 Estimation Ignoring Endogeneity

If we ignore the endogeneity of expenditure and the unobserved candidate characteristics, we can ignore the presence of the unobserved candidate characteristics and estimate the

⁵The instruments we use will be discussed in the data section.

⁶This argument will be strengthened once we are able to gather data on each candidates gender, age, and race and include those variables in the estimation. This data gathering is currently happening.

model using nonlinear least squares. The estimation equation is:

$$(\alpha, \beta) = \min_{\alpha, \beta} \frac{1}{2M} \sum_{m=1}^M \sum_{j \in \{0,1\}} \left[s_{mj} - \sum_{n \in \{0,1,N\}} \mu_{mn} * \frac{e^{\delta_{mnj}}}{1 + \sum_{j \in \{0,1\}} e^{\delta_{mnj}}} \right]^2 \quad (11)$$

with $\delta_{nmj} \equiv \alpha_n \ln(p_{mj}) + \beta_n x_{mj}$

This estimation method picks the vector (α, β) so that the sum of square differences between the observed vote share and the predicted vote share for each candidate in each district is as small as possible. Robust standard errors are then calculated following Wooldridge (2002).

2.7.2 Estimation Controlling for Endogeneity

Since the estimation of this type of model is discussed extensively in BLP and in many other sources, we provide only a brief summary of the process. The endogeneity of the unobserved characteristics provides the standard instrumental variables moment conditions for the model, $E(Z'\xi) = 0$. Z is a vector of exogenous covariates, including instruments for the endogenous expenditure regressors and ξ is the vector of unobservable characteristics for all the candidates. The main result from Berry (1994) is that for each vector of parameters (α, β) , there exist a unique vector ξ that equates the predicted vote share function with the observed vote shares, $s = \hat{s}_j(\alpha, \beta, \xi; \mu)$, in every market. This means that ξ is a highly nonlinear function of $(\alpha, \beta; s, \mu)$, where s is the vector of observed market shares. The ξ_j for each candidate can be found market by market for each candidate using a contraction mapping similar to BLP⁷. Therefore, the estimation of (α, β) reduces to a standard GMM objective function.⁸

⁷Mike Carnall graciously provided an outline of how to program this contraction mapping with discrete heterogeneity. Also, Dube et al (2009) provides great guidance on practical problems of this estimation technique.

⁸Our problem is computationally much simpler than BLP because it does not require any simulation of integrals to estimate the coefficient distribution parameters. A copy of the Matlab code used to estimate

$$(\alpha, \beta) = \min_{\alpha, \beta} [Z'\xi(\alpha, \beta; s, \mu)]'[Z'Z]^{-1}[Z'\xi(\alpha, \beta; s, \mu)] \quad (12)$$

The standard errors for this are calculated using the delta method.⁹

3 Data

3.1 Overview

Our data set consists of US Congressional district-level voter registration statistics by party, the party of incumbency and candidate expenditures for the 2002, 2004, 2006 and 2008 Congressional House elections. The data set was compiled from different sources. The voter registration statistics and final vote counts were collected from the each state’s Board of Elections. Some states provide this information on their website and some had to be contacted for the information. All of the incumbency and expenditure data come from the Federal Election Commission.

There are 30 “states”¹⁰ that require voters to register for a specific party if they want to vote in the primary elections. These states are said to have “closed primaries”. We do not include states that have “open primaries” because they either do not ask for party affiliation on their voter registration forms or the chosen party has no implications for their voting options. Therefore, the states with “open primaries” do not provide a reliable report of the voter registration statistics. Of the 30 states with “closed primaries”, we were able to assemble data from from the Congressional districts in 26 states. The majority of these districts have voter registration data available for all four elections this decade¹¹. We limit our model is available upon request.

⁹We do not need to correct for measurement error in our standard errors since the observed vote shares are calculated using the actual number of votes received.

¹⁰29 states plus the District of Columbia

¹¹The data for districts in Kansas and Utah is only available from 2008 and Connecticut does not have the information for 2006.

the data to the elections this decade because the number of Congressional districts per state is reallocated after every census. For example, there are Congressional district from the 2000 election that were erased for the 2002 election and vice versa. Also, we do not employ data on districts from states that lack the voter registration statistics aggregated at the congressional district level. The data for the missing districts could conceivably be aggregated by contacting each individual voting precinct in each congressional district, but this would require contacting thousands of different precincts across many states and lies beyond the scope of our study.

The theoretical model is designed to analyze the districts where individuals have the choice between voting for a Republican candidate or a Democratic candidate. This leads us to include only districts in which both a Republican and a Democratic candidate ran for the office and spent more than \$10,000 each and to exclude districts where any additional candidates ran and spent more than \$10,000. As mentioned earlier, this restriction is based on the fact that every member of the US House of Representatives is either a Republican or a Democratic. In the US, the two parties completely dominate the federal government and voting for a candidate outside of these two parties is similar to not voting.

Given these restrictions, our sample observation size is 478 congressional districts. With 2 candidates in each district, this yields 956 different observations¹². For example, the 2nd Congressional district in California during the 2002 election is one district with 2 observations, while the 2nd Congressional district in California during the 2004 election is another district with 2 different observations. The summary statistics for the congressional districts is provided in the Table 1. All expenditure data is in 2008 dollars.

The summary statistics show that, on average, about 77% of the registered voters in our districts are partisan to either the Democratic Party or the Republican Party and that a

¹²This sample size will be reduced in the instrumental variables estimation because some of our instruments are lagged values.

Table 1:

Summary Statistics of Congressional Districts

Variable	Full Sample		Restricted Sample	
	Mean	St. Dev.	Mean	St. Dev.
Pct. Registered Democrat	.4494	.1382	.4082	.1133
Pct. Registered Republican	.3326	.1226	.3652	.0990
Pct. Democrat Incumbent	.44744	-	.3808	-
Pct. Republican Incumbent	.4322	-	.4979	-
Democratic Expenditure	905,247	973,043	1,057,618	1,066,246
Republican Expenditure	855,201	1,042,171	1,136,768	1,130,382
Democratic Vote Share	.2946	.1511	.3058	.1122
Republican Vote Share	.2524	.1500	.2964	.1099
Districts	782		478	

great deal of money is being spent in an attempt get elected. Candidates from both parties' average over a \$1,000,000 in campaign expenditures. There is a very high variance on the amount spent, but overall these elections consume a large amount of financial resources. Last, the vote shares are calculated by dividing the number of votes received by the total number of registered voters in the district. Both parties average about a 30% vote share, which corresponds to a 60% voter turnout rate, which is fairly typical in the US.

Table 1 provides summary statistics for the full sample of 738 districts along with the restricted sample that will be used for estimation. There is no statistical difference between the two samples along these dimensions, however the average expenditure measure is larger in the restricted sample because one of the restrictions involves removing districts having low expenditure by one of the candidates. Along with higher expenditures, the data restrictions

cause the difference between Pct. Registered Democrats and Republicans to be smaller. The averages in the restricted sample are only 4% different, whereas they were 11% different in the full sample. The higher expenditures and lesser difference in registered voters show that the the restricted districts are much more likely to have competitive races. As argued in Erickson & Pelrey (2000), this leads to much better estimates of the effect of expenditures on Congressional outcomes.

3.2 Instruments

For a valid instrument in this situation, we need a regressor that is correlated with the candidate's expenditure but uncorrelated with the number of votes that candidate received in the election. Here we exploit the iterative nature of elections and the fact that Congressional district boundaries provide a clear distinction for each market. United States congressional elections occur every two years. Therefore, we can use characteristics of lagged elections and characteristics of Congressional districts within the the same state to instrument for candidate expenditure.

Since we need to identify coefficients for three different types of agents, we need three different instruments. We start by adapting the same two instruments that Rekkas (2007) used. The first instrument is the total expenditures by the Democratic candidate and the Republican candidate from the previous election. This instrument was first used by Gerber (1998) in his study of Senate elections. The second instrument is a measure of how close the previous election was in each district. This instrument can be interpreted as the historical competitiveness of the election for each district. It is defined as the absolute value of the difference in percentage of votes received by the Democratic and Republican candidates.

Therefore, the closeness measure ranges from 0 to 1, where the values near zero are districts where the previous election results were very close and the values near 1 are where the previous election results were landslides. The third instrument is the average expenditure of all the candidates in the same state during the same election. Since state and congressional district boundaries are clearly defined, this instrument uses information from the districts that are related by being part of the same state¹³.

The use of lagged total expenditure as an instrument, a la Gerber (1998) and Rekkas (2007), is less obvious in our environment. The main issue with using the lagged total expenditure is that congressional elections in the United States are different from the Senate elections that Gerber studied and Canadian parliamentary elections Rekkas studied. Senate elections only occur every six years and Canadian elections occur every four years. In their cases, it is easy to assert that previous election expenditures are uncorrelated with the current campaign elections. Both authors argue that lagged total expenditure is valid because they use the total expenditure instead of just the previous candidates expenditure and there is very large turnover between incumbents and challengers. This characteristic is not as straightforward for US congressional elections. However, unlike Gerber (1998) and Rekkas (2007), we are able to make use of the voter registration statistics of each district. Since these statistics change over time, the voter registration statistics act partially as a summary of past candidate expenditures. Since a lot of campaign expenditure is concentrated on registering voters to their party, the correlation between lagged expenditure and current expenditure can be captured in the registration statistics. This fact, combined with the use of total lagged expenditure instead of individual candidate's lagged expenditure,

¹³See Appendix A for the reduced form regressions of candidate expenditure on the instruments.

makes it a valid instrument.

4 Empirical Results

The results of three different specifications of the theoretical model are presented in Tables 2 and 3. Both tables show three different specifications of the model. The first column does not interact expenditure with any other variables. The second column interacts expenditures with the incumbency dummy variable to estimate the differing effects of spending by incumbents and challengers. The third column interacts expenditure with the Democratic party dummy variable. This allows us to differentiate the effect of spending by the Democratic candidates and the Republican candidates. Specifically, it allows us to estimate of the effect that expenditure by the Democrat has on registered Democrats, Republicans and nonpartisans. The same holds true for expenditure by a Republican candidate.¹⁴ One surprising result from Rekkas(2007) was that the variance of the expenditure coefficient distribution was larger than the mean, which implies that not all voters have a positive marginal utility for expenditure. As will be shown, our results confirm this fact and show that it is a result of registered Democrats reacting negatively to Republican candidates and registered Republicans reacting negatively to Democrat candidates.

As mentioned earlier in the paper, the following tables show the coefficient results from two different estimation methods. All specifications also include dummy variables for the State of each district, but these coefficients were omitted since they are not of interest. The first table presents estimation results when it is assumed that the unobserved product

¹⁴We only allow for random coefficients on the expenditure covariate at this time because we need to identify more instruments before allowing random coefficients on Incumbency and dDem

characteristics are uncorrelated with any of the regressors. In the this situation, there is no endogeneity bias by ignoring these unobserved characteristics so the model can be estimated using Nonlinear Least Squares (NLS) and the resulting coefficients are reported in Table 2. The second table presents the estimation results when we control for the endogeneity of the unobserved product characteristics. The estimation is done using the BLP GMM methodology. Each estimation also reports one or two Wald statistics that test if the random coefficients are equal statistically different. These tests are discussed in the subsection 4.3. The last section presents the average elasticities predicted by these results and breaks the elasticities into agent specific elasticities.

4.1 Model without Endogeneity

Before examining the the random coefficients on expenditures, first look at the the coefficient results for the other control variables. Care must be given when interpreting these coefficients since they are not coefficients from a linear model. These coefficients can be interpreted in an analogous manner to a standard logit regression. The Senate Race, Presidential Race, and Constant regressors are the same across the two candidates so they will only effect the probability of voting. The coefficient terms support the argument that a person is predisposed not to vote because there is a time cost to voting. The Constants are all negative and this means that without any other factors, an individual would be likely not to vote. If all other factors were equal to zero, the negative constant value of -1.65 implies that each type of agent would have a 14% probability of voting. This substantiates the idea that there is implicit cost of voting. Without other factors, a person is unlikely to take the time to vote.

Table 2:

NLS Parameter Estimates

VARIABLES	(1)	(2)	(3)
$(\mu_0)\ln(\text{expend})$	0.1793*** (0.0123)	0.07122*** (0.0298)	0.0018 (0.0324)
$(\mu_1)\ln(\text{expend})$	0.1348*** (0.0136)	0.2224*** (0.0235)	0.2438*** (0.0203)
$(\mu_N)\ln(\text{expend})$	0.1358*** (0.0146)	0.1697*** (0.0215)	0.1114*** (0.0227)
$(\mu_0)\ln(\text{expend})*\text{Incumb}$		-0.0802*** (0.0344)	
$(\mu_1)\ln(\text{expend})*\text{Incumb}$		-0.2749*** (0.0326)	
$(\mu_N)\ln(\text{expend})*\text{Incumb}$		-0.1702*** (0.0250)	
$(\mu_0)\ln(\text{expend})*\text{dDem}$			0.1805*** (0.0378)
$(\mu_1)\ln(\text{expend})*\text{dDem}$			-0.2001*** (0.0425)
$(\mu_N)\ln(\text{expend})*\text{dDem}$			0.0783*** (0.0293)
Incumbent	0.2603*** (0.0027)	1.1086*** (0.0024)	0.2702*** (0.0027)
dDem	0.0554*** (0.0004)	0.0401*** (0.0004)	0.0179*** (0.0004)
Senate Race	-0.0172*** (0.0021)	-0.0128*** (0.0020)	-0.0313 (0.0020)
Presidential Race	0.7882*** (0.0014)	0.7890*** (0.7890)	0.7974*** (0.0014)
Constant	-1.6542*** (0.0739)	-1.6528*** (0.0652)	-1.4696*** (0.0760)
Observations	956	956	956
R^2	0.80	0.83	0.81
Wald Tests:			
$\chi^2_{\ln(\text{expend})} =$	5.8871	10.154	25.811
Prob =	0.9473	0.9938	1.0000
$\chi^2_{\ln(\text{expend})*\text{Interaction}} =$		9.188	28.8906
Prob =		0.9899	1.0000

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Note: Coefficients for the state fixed effects are omitted.

The positive coefficients on Presidential Race show that a person is more likely to vote in the Congressional elections when they can also a Presidential election at the same time. The estimate is very consistent throughout the specifications and the estimate implies a marginal effect of about 9%. A little surprising the result that the Senate Race coefficient is significantly negative. This implies that voters are less likely to vote in a Congressional election when a Senate Race is running concurrently. However, the coefficients are all very small, with a marginal effect of -.3%, and therefore this has very little impact. A concurrent Presidential election increases the probability of voting much more than a concurrent Senate race decrease the probability. Overall, these effects fit within standard theories of voting.

The Incumbent and Democratic dummy variable regressors do vary by candidate and therefore effect the choice of who to vote for. The dDem coefficients are all positive but very small. The estimates imply that a Democratic candidate has about .25% advantage in vote share. However, an Incumbent has a substantial intrinsic advantage. The first and third columns, without the expenditure Incumbency advantage, both estimate that an incumbent has about a 3% point advantage in an election. This estimate corresponds exactly with the prior estimates by Levitt (1994). The second column implies an advantage of 13%. This seems very high in that and we believe this is a result of not controlling for Incumbent performance while in office that is included in the unobserved candidate characteristics.

Now, lets look at the random expenditure coefficients and how to interpret them. The first specification does not interact the expenditure term and therefore the coefficient on $(\mu_0)\ln(\text{expend})$ is the average marginal effect of expenditure on a registered Democrat. In the second specification, $\ln(\text{expend})$ is interacted with the Incumbency dummy variable, so $(\mu_0)\ln(\text{expend})$ is now the average marginal effect of expenditure on a registered Democrat

by a non incumbent candidate. The average marginal effect of an incumbent candidate on a registered Democrat is the sum of $(\mu_0)\ln(\text{expend})$ and $(\mu_0)\ln(\text{expend})*\text{Incumb}$. Finally, the third specification interacts expenditure with the Democratic party dummy variable. $(\mu_0)\ln(\text{expend})$ is now the average marginal effect of expenditure on a registered Democrat by a Republican candidate. The average marginal effect of Democratic candidate on a registered Democrat is the sum of $(\mu_0)\ln(\text{expend})$ and $(\mu_0)\ln(\text{expend})*\text{dDem}$. Similarly to the (μ_0) coefficients, the $(\mu_1)\ln(\text{expend})$ coefficients are the effect on a registered Republican and the $(\mu_N)\ln(\text{expend})$ coefficients are the effect on a nonpartisan registered voter.

The signs of the coefficients are across all the specifications are as expected. In the second specification, the results are a little surprising that all the effects of incumbent expenditure are estimated to be negative which implies an incumbent loses vote share by expending more. However, all these coefficients are not significantly different from zero and this in line with previous estimates of incumbency effects. The first specification shows that all types of registered voters will respond to an increase in expenditure. The third specification then breaks this result down and shows that different types of agents respond very differently to the two types of candidates. The estimates show that a Democrat has zero (0.00018 but insignificant) marginal response to expenditure by a Republican but a relatively large response to the expenditure by a Democratic candidate.¹⁵ Conversely, a registered Republican has a very large marginal response to expenditure by a Republican (0.2438) but a relatively small response to expenditure by a Democratic candidate ($0.2438 + -0.2001 = 0.437$). Lastly, the estimate results show that expenditure by either candidate increase

¹⁵The magnitude and effects of these coefficients will be discussed in a more intuitive way in the elasticity subsection.

the probability that a non-partisan voter will vote for him, with a higher responsiveness occurring for the Democratic candidate.

4.2 Model with Endogeneity

The BLP model is estimated using lagged total expenditure, average expenditure per candidate in the rest of the state and lagged closeness as instruments, and the results are presented in 3. The coefficient estimates for the nonexpenditure control variables are all very similar to the the NLS estimates except for Incumbency dummy variable in the second specification. The magnitude and marginal effect are now in line with the rest of the specifications however this specification is the least precisely estimated of all six specifications. This provides an indication that Congressional performance while in office is a very important unobserved candidate characteristic. While the estimates for non incumbents remain precise, the Incumbent estimates are have very high standard errors.

While the estimates on the control variables are all similar to the NLS results, the estimates on the expenditure coefficients change dramatically. In the first specification, the NLS results all had similar, positive average marginal effects. The BLP results show that non-partisans are not affected on the margin, while registered Republicans have a much higher responsiveness than registered Democrats. The second specification implies that a non-incumbent can gain more Republican votes but loses Democratic and non-partisan votes with an increase in expenditure. However, the imprecision in these estimates make this second specification difficult to interpret. Finally, the third specification provides results that are very similar to the NLS results. A Republican candidate can increase the chance a registered Republican and non-partisan votes for him while decreases the the chance a

Table 3:

BLP Parameter Estimates

VARIABLES	(1)	(2)	(3)
$(\mu_0)\ln(\text{expend})$	0.1296*** (0.0418)	-0.2322*** (0.0147)	-0.1444** (0.0692)
$(\mu_1)\ln(\text{expend})$	0.3224*** (0.0644)	1.0129*** (0.1177)	0.2713*** (0.0779)
$(\mu_N)\ln(\text{expend})$	0.0426 (0.1033)	-0.3327*** (0.0120)	0.1994*** (0.0407)
$(\mu_0)\ln(\text{expend})*\text{Incumb}$		0.4898*** (0.1422)	
$(\mu_1)\ln(\text{expend})*\text{Incumb}$		-0.5541 (0.4385)	
$(\mu_N)\ln(\text{expend})*\text{Incumb}$		0.1542 (1.0216)	
$(\mu_0)\ln(\text{expend})*\text{dDem}$			0.3509*** (0.0709)
$(\mu_1)\ln(\text{expend})*\text{dDem}$			-0.0888 (0.3208)
$(\mu_N)\ln(\text{expend})*\text{dDem}$			-1.8176*** (0.2238)
Incumbent	0.2013** (0.0128)	0.2462 (0.7948)	0.3255*** (0.0388)
dDem	0.1457*** (0.0248)	-0.0789* (0.0440)	0.2899*** (0.0882)
Senate Race	-0.0285* (0.0162)	0.0236 (0.0191)	-0.0219 (0.0885)
Presidential Race	0.8244*** (0.0464)	1.2099*** (0.0723)	0.8147*** (0.054)
Constant	-1.7448*** (0.3075)	-2.3741*** (0.1093)	-1.6220*** (0.1529)
Observations	728	728	728
Wald Tests:			
$\chi^2_{\ln(\text{expend})} =$	4.3446	277.77	21.844
Prob =	0.8861	1	1
$\chi^2_{\ln(\text{expend})*\text{Interaction}} =$		58.072	3949.5
Prob =		1	1

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Note: Coefficients for the state fixed effects are omitted.

Democrat votes for him by expending more. However, these results say that a Democratic candidate will increase positively effect both registered Democrats and Republicans, but greatly reduce the the probability a registered non partisan will vote for him. The magnitude of the $(\mu_N)\ln(\text{expend})^*\text{dDem}$ coefficient is very large and more research is needed to explain this¹⁶.

4.3 Testing the Random Coefficients

All previous literature except for Rekkas(2007) is actually a reduced form of our model. The estimates implicitly assumed that expenditure affected the all voters in the same way. We explicitly model the heterogeneity and therefore are able to test this assumption. The Wald test statistic at the bottom of each column in the results table is a test of the null hypothesis that the three discrete random coefficient values are equal to each other. For example, in Table 2, column 2, the first Wald statistic tests $H_0 : (\mu_0)\ln(\text{expend}) = (\mu_1)\ln(\text{expend}) = (\mu_N)\ln(\text{expend})$. The second Wald statistic tests $H_0 : (\mu_0)\ln(\text{expend}) * \text{Incumb} = (\mu_1)\ln(\text{expend}) * \text{Incumb} = (\mu_N)\ln(\text{expend}) * \text{Incumb}$. The null hypothesis is rejected at the 10% level in all the specifications except in the BLP specification without an interaction. However, this test has a p-value of 0.8861. All of these test provide very strong evidence that the random coefficients are important because different types of voters respond in very different ways to candidate expenditures, and this is the first

¹⁶As stated earlier, the expenditure coefficient results from the BLP estimation are very different from the NLS estimation and are sometimes counterintuitive. However, the NLS results are able to replicate the results of previous empirical work without controlling for endogeneity and provide very intuitive results. Therefore, we are working on an endogeneity test to see if using the observed heterogeneity of voters removes the endogeneity of candidate expenditures.

Table 4:

NLS Elasticities		
	1% Increase in Expenditure by:	
	Candidate 0 (Democrat)	Candidate 1 (Republican)
$(\Delta \hat{s}_0)$	0.1061	-0.0458
$(\Delta \hat{s}_1)$	-0.0475	0.1078
$(\Delta \hat{s}_N)$	-0.0469	-0.0452

Values are the average elasticities from all the districts.

Table 5:

BLP Elasticities		
	1% Increase in Expenditure by:	
	Candidate 0 (Democrat)	Candidate 1 (Republican)
$(\Delta \hat{s}_0)$	0.1252	-0.0651
$(\Delta \hat{s}_1)$	-0.0659	0.1265
$(\Delta \hat{s}_N)$	-0.0491	-0.0487

Values are the average elasticities from all the districts.

paper to precisely estimate the effects.

4.4 Expenditure Elasticities

Magnitude of a marginal increase in expenditure is difficult to interpret through the estimated coefficients. Therefore, Tables 4 and 5 show the average own and cross elasticities of an increase in expenditure by each candidate. These elasticities were calculated using specification (1) of each estimation. The values in each table show average percentage increase in vote share for each candidate by if he would increase his expenditure by 1%. The average expenditure in these elections was about \$1,000,000, so 1% is about \$10,000. These tables show that an Democrat or Republican can increase his vote share by about .11%

by spending 1% more. There are 400,000 voters on average in each district, so a \$10,000 increase gets a candidate 440 more votes. This increase comes from switching voters from the other candidate and also convincing people who would otherwise abstained to vote.

5 Conclusion

This is the first paper to use a discrete choice model when the population heterogeneity is observed. We provide new insights on the effect that candidate characteristics have on election outcomes and on the response to these characteristics by different groups. Our results highlight a very important fact that has not been addressed by previous literature: the distribution of registered voters in each district is key to the election results. This paper quantifies the fact that registered voters respond to a larger degree to candidates from their same party. This finding is important result because individual states have the power to redraw their congressional districts at anytime. When new congressional districts are created with the intent to favor one party at the expense of the other, this strategy is known as redistricting. These results quantify the fact that districting can be a very powerful tool in affecting the outcome of congressional elections. Combined with the relatively low marginal gains to campaign expenditures, campaign policy reform for congressional elections should be more focused on ensuring the fair drawing of districts rather than reforming the laws of campaign expenditures.

6 Appendix

A Reduced Form Regression on Expenditures

This table shows that the instruments satisfy the first condition for being valid instruments. The table shows that the instruments are all individually highly correlated with the $\ln(\text{expend})$ covariate controlling for all the other covariates. Then, column 4 shows that the correlation still exists for each instrument when all the instruments and all the other covariates are included in the regression. The signs of most of the correlations are as expected also. Higher expenditure in the previous election is correlated with higher expenditure in the current election. Lagged Closeness ranges from 0 to 1 where a value of 0 means the the prior election was a tie and 1 where one candidate received all the votes. The negative correlation implies that as the close outcomes of the prior election are correlated with more expenditure in the current election. The sign of the correlation between $\ln(\text{restAvg})$ and $\ln(\text{expend})$ is not as easy to explain. The negative correlation implies that as the larger average expenditure of candidates from the rest of the state is correlated with lower expenditure levels. While this does not have an intuitive explanation, the correlation, and all the other correlations, are highly significant and that is all that is required to satisfy the first condition of a valid instrument.

Reduced Form Regressions				
VARIABLES	(1)	(2)	(3)	(4)
	ln(expend)	ln(expend)	ln(expend)	ln(expend)
ln(Lagged Total Expend)	0.585*** (0.0662)			0.337*** (0.0841)
Lagged Closeness		-1.714*** (0.205)		-0.906*** (0.247)
ln(restAvg)			-0.660*** (0.145)	-1.317*** (0.236)
Incumbent	1.544*** (0.0857)	1.539*** (0.0869)	1.507*** (0.0818)	1.569*** (0.0849)
dDem	0.188** (0.0929)	0.188** (0.0936)	0.132 (0.0879)	0.200** (0.0915)
Presidential Race	-0.201** (0.102)	-0.209** (0.102)	-0.0926 (0.0899)	-0.532*** (0.119)
Senate Race	-0.0341 (0.0987)	-0.0453 (0.0979)	0.00623 (0.0924)	-0.305*** (0.112)
Constant	0.922 (0.577)	4.604*** (0.347)	6.608*** (0.981)	11.77*** (1.976)
Observations	746	746	932	728
R^2	0.381	0.366	0.295	0.417

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Note: Coefficients for the state fixed effects are omitted.

B IV Results Matching Previous Work

This appendix shows the results of the estimation if we make the assumption that there are not any heterogenous agents in the Congressional districts, which is a special case of our model. These are presented to show that are data presents very similar results to the previous empirical work on estimating candidate effects on vote shares using instruments to control for endogeneity. The estimation was done using Two Stage IV estimation and using all three instruments.

IV Parameter Estimates

VARIABLES	(1)	(2)	(3)	(4)
ln(expend)	0.175*** (0.0238)	0.248*** (0.0246)	0.182*** (0.0316)	0.264*** (0.0372)
ln(expend)*Incumb		-0.232*** (0.0513)		-0.237*** (0.0545)
ln(expend)*dDem			-0.0122 (0.0441)	-0.0522 (0.0507)
ln(expend)*Incumb*dDem				0.00335 (0.0189)
Incumbent	0.214*** (0.0424)	1.206*** (0.227)	0.213*** (0.0423)	1.234*** (0.229)
dDem	0.110*** (0.0257)	0.0931*** (0.0254)	0.153 (0.176)	0.284* (0.168)
Senate Race	-0.0371 (0.0293)	-0.0284 (0.0293)	-0.0423 (0.0293)	-0.0386 (0.0284)
Presidential Race	0.757*** (0.0291)	0.770*** (0.0295)	0.751*** (0.0283)	0.759*** (0.0286)
Constant	-2.084*** (0.136)	-1.635*** (0.101)	-1.424*** (0.121)	-1.692*** (0.143)
Observations	728	728	728	728
R^2	0.717	0.710	0.717	0.718
Endogeneity Test: $\chi^2 =$	2.919	17.62	3.659	16.96
Prob =	0.0876	0.000149	0.160	0.00197
Over ID Test: $\chi^2 =$	1.243	9.149	5.656	12.42
Prob =	0.537	0.0575	0.226	0.133

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

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