**ECONOMICS 6331, Fall 2003**
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**Midterm Exam 2 —6 questions. All sub-questions carry equal weight. You need to show how you arrive at your conclusions—answers like "yes" or "no" without further elaboration will get 0 points.)**

1. (24%) Consider two random variables X and Y. Assume they both are discrete and that both X and Y can take the values 1,2, and 3. The probabilities for (X,Y) are shown in the following table:

| | X=1 | X=2 | X=3 |
|---|---|---|---|
| Y=1 | 2/24 | 3/24 | 7/24 |
| Y=2 | 1/24 | 3/24 | 2/24 |
| Y=3 | 1/24 | 2/24 | 3/24 |

i) Find the marginal probabilities of X and Y. Mark clearly which are the marginal probabilities of X and which are the marginal probabilities of Y. Explain what the marginal probabilities measure.
ii) Find the mean and the variance of Y.
iii) Are the events X = 1 and Y = 1 independent events?
iv) Are the random variables X and Y independent?
v) Find the probability $P(\{X > 2\} \bigcap \{Y \leq 2\})$
vi) Find the conditional distribution of $X$ given $Y = 2$.
vii) Find the random variable $E(X|Y)$.
viii) Find $Var(X|Y = 2)$.

2. (16%) Consider the density $f(x, y) = 2;\ \ 0 < y\ \ < x\ \ < 1$. Find the bivariate CDF $F(x, y)$. (Hint: treat the cases $x < y$ and $y < x$ separately).

3. (20%) Assume $X \sim N(1, 4)$, $Y \sim N(2, 9)$, and the covariance between $X$ and $Y$ is 1.
i) What is $E(X|Y = 2)$? (State the formula you use and then the number.)
ii) What is $Var(X|Y = 3)$?

4. (10%) If $X$ and $Y$ are jointly normally distributed and the marginal distribution of both $X$ and $Y$ are standard normal and $X$ and $Y$ are independent, what is the value of the joint CDF at the point $(0, 0)$? (I.e., what is $F(0, 0)$?)

5. (15%) Prove the law of iterated expectations (you can do the discrete or the continuous case).

6. (15%) If $X$ and $Y$ are jointly normally distributed, derive the conditional distribution of $X$ given $Y$. (To cut down on clutter, you may—if you so choose—assume that both $X$ and $Y$ have mean 0.)