

**Midterm Exam 2, April 6—5 questions. All sub-questions carry equal weight except where otherwise indicated.**

1. (20%) Assume that a representative agent has a utility function

$$U(C, L) = C^2 - L^3 .$$

(The convex shape of  $C^2$  is unusual, but it is not a typo.) Assume that agent  $i$  supplies output  $Q_i$  produced by the production technology  $Q = L$ . The agent sets the relative price  $P_i/P$ , where  $P$  is the aggregate price index (assume there are many types of agents so a change in  $P_i$  doesn't change  $P$ ) and faces a demand function

$$Q_i = Y \left( \frac{P_i}{P} \right)^{-2} .$$

- a) Find the agent's supply function when the agent takes  $P$  as given.
- b) Find the equilibrium level of output in the economy.
- c) What level of output would a social planner choose?

2. (30%) Assume output in an economy is determined by the equilibrium condition that aggregate demand  $E$  is equal to total output  $Y$ . Assume that  $E = C + I + G$  where consumption  $C = 2 + .6*(Y - T)$  ( $T$  is net taxes), investment is exogenous at 1, and government consumption is exogenous at 2.  $T$  equals 2. Assume that inflation and expected inflation is 0 and that  $P = 1$ . Money supply is exogenous at 10, and the demand for money is  $P * L(Y, r)$  where  $L(Y, r) = Y - 0.1 * r$ . (The numbers are not chosen to give reasonable values for the solution, so don't worry about "crazy" interest rates etc.)

- a) Derive the IS curve (meaning give the equation with the actual intercept and slope implied by the numbers given).
- b) Solve the model for the equilibrium level of output and interest rate.
- c) What happens if  $M$  doubles to 20? (Find the new level of output and interest rate.)
- d) Assume that government consumption and net taxes both double to 4. What is the effect on output?

3. (20%) This is about the CAPM. Assume the market return has a standard deviation of 0.2, Consider three stocks a, b, and c. Stock  $a$  has mean return  $Er^a = .1$ , standard deviation of returns  $std^a = .2$ , covariance with the market return 1.0; stock  $b$  has mean return  $Er^b = 0.05$ , standard deviation of return  $std^b = 0.2$ , and covariance with the market return 0.5, stock  $c$  has mean return  $Er^c = 0.0$ , standard deviation of returns  $std^c = .8$ , covariance with the market return 0.0.

- a) What is the return to the safe asset?
- b) What is the return to the market portfolio?

4. (15%) a) What is meant by “efficient markets” (semi-strong form, in the sense of Fama).  
 b) Assume markets are efficient, as in part a, and assets are priced with a pricing kernel. Find the pricing kernel.  
 c) What is the relation between the safe rate of interest and the pricing kernel? (This question is independent of question b.)
5. (15%) The consumer maximizes  $\sum_{t=0}^{\infty} 0.9^t E_0 U(C_t)$ . Consider the case with utility is exponential  $U(C) = -\exp(-C)$  and  $C_t$  is distributed i.i.d. (independently, identically, distributed)  $N(\mu, \sigma^2)$ . .
- a) Assuming the Euler equation holds, what is the (period 0) price of a one-period discount bond?  
 b) What is the price of a two-period discount bond?
- Now assume that utility is quadratic  $U(C) = C - \frac{1}{200}C^2$
- c) What is the price of a one period discount bond?  
 d) Give one reason, in terms of concepts from class, why the price is now different.