

Midterm Exam 2, April 2—4 questions. All sub-questions carry equal weight except where otherwise indicated.

1. (20%) Consider the CAPM. Assume that there are two states of the economy next year, “A” and “B”; state A happens with probability 0.2 and state B with probability 0.8. Assume the return to the market portfolio is 5% in state A and 10% in state B. Now consider assets 1 and 2. For these we know the pay-outs. For asset 1 the payout is 40 in state A and 60 in state B, while for asset 2 the payout is 20 in state A and 80 in state B. Use the CAPM as it was derived in the handout. The safe rate of return is 1%.

a) What would be the rate of returns of assets 1 and 2?

Now assume that in state A the return to the market portfolio is 10% and in state B the return to the market portfolio is 0%.

b) Find the rate of returns to asset 1 and asset 2 under this assumption.

2. (50%) a) Derive a relation between excess returns and consumption growth under the assumption that utility is logarithmic. (I want you to replicate the steps in the derivation of the Consumption-CAPM, except that you should stop before the step involving the consumption β .)

Now assume that an agent lives for 2 periods in an economy with perfect Arrow-Debreu markets and no storage. Assume that the endowment of the agent is $y_0 = 2$ in period 1 his or her endowment is $y_1^g = 3$ in the “good state” g . In the “bad state” b the endowment of the agent is $y_1^b = 1$. Assume that the good state happens with probability $1/2$. Assume the price of the Arrow security which pays out in the good state is $\frac{10}{33}$ and the price of the Arrow security which pays out in the bad state is $\frac{20}{33}$.

Assume each agent maximizes a utility function

$$U(C_0) + E_0 U(C_1) .$$

where $U(C) = \log C$.

b) Find the safe rate of interest r .

c) Find the level of consumption of the agent in periods 0 and 1 and both states of the world.

d) Interpret the relation between consumption in the good and bad state (why is one higher than the other) and between consumption in the initial and the second period. (You should be able to do that even if you could not numerically solve for them in the previous question.)

e) An asset has a payout of 10 in the good state and 5 in the bad state. Find the average rate of return on this asset.

Now assume that the agent does not have access to Arrow-Debreu securities but can trade in a

risk-less bond. Assume that the interest rate is unchanged. (If you didn't find it in question b, you will use an assumed rate.)

f) Write down one equation in one unknown in the amount (B) of the bond purchased.

g) Would B be positive or negative?

3. (10%) Consider the CAPM-model.

Assume the world only have two outcomes ("states of the world"). Let X be an asset whose payout PO_X is 100 if "shine," a situation where the market return is 10 percent. "Shine" has probability 0.5. If "rain," the pay-out to the asset is 50 and the market return is 5 percent; "rain" also has probability 0.5. Assume that the safe rate of interest is 1 percent.

a) What is the expected return (ER_X) to an investment in X?

4. (20%) a) Explain carefully the concept of (semi-strong) efficient markets (as defined by Fama).

b) Explain the relation between the Euler equation and the concept of efficient markets.