

Midterm Exam 2, April 1—6 questions. All sub-questions carry equal weight except where otherwise indicated.

1. (15%) Assume that inflation in period t is I_t and I_t is positively correlated with the (gross) market return R_t^M . Inflation has a positive mean (i.e., different from 0). Set the safe rate of interest to 0 for simplicity. A researcher may find the following patterns (with significant positive coefficients) for assets i :

$$\begin{aligned} (A) \quad R_t^i &= \alpha^i I_t + u_{it} , \\ (B) \quad R_t^i &= \beta^i R_t^M + u_{it} , \\ (C) \quad R_t^i &= \alpha^i I_t + \beta^i R_t^M + u_{it} , \end{aligned}$$

where the u error terms are white noise.

- i) In case A, does the CAPM hold? does the CAPM not hold? or, we cannot tell?
- ii) In case B, does the CAPM hold? does the CAPM not hold? or, we cannot tell?
- iii) In case C, does the CAPM hold? does the CAPM not hold? or, we cannot tell?

2. (20%) Consider a two period model with periods 0 and 1 and states-of-the-world A and B in period 1. Assume the probability of state A is 30 percent and that a full set of Arrow securities exist. An agent maximizes a utility function

$$\log(C_0) + E_0 \log(C_1) .$$

Assume that you know that consumption C_0 is 10 units while consumption in period 1 is $C_1^A = 20$ and $C_1^B = 10$.

- i) Find the (period 0) price of the Arrow securities for each of the states of the world.
- ii) Find the safe rate of interest (if you give one equation in one unknown that is sufficient if you also point out if the interest rate is positive or negative).

3. (10%) Consider the CAPM-model.

a) An asset whose payout $PO_X = X$ with probability 0.5 and $PO_X = 0$ with probability 0.5. Assume the return r_X has a covariance with the market return (measured in percent) of 20 and that the variance of the market return is 5 (measured in percent). The expected rate of return of the market is 5 percent. Assume that the safe rate of interest is 2 percent. Finally assume the price of the asset is 10.

- a) What is the expected return ($E r_X$) to an investment in X?
- b) Find X .

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4. (20%) Assume a representative agent lives of three periods (period 0, 1, and 2) and that he or she can trade in 1- or 2-period bonds.

Assume that the agent maximizes

$$U(C_0) + \frac{1}{1.1} E_0 U(C_1) + \frac{1}{(1.1)^2} E_0 U(C_2) ,$$

where $U(C) = \log C$.

Assume $C_0 = 10$, C_1 is 9 with probability 0.5 and 11 with probability 0.5, while C_2 is independent of C_1 and equals 5 with probability 0.5 and 15 with probability 0.5.

i) What is the safe rate of interest from period 0 to period 1?

ii) What is the forward rate of interest from period 1 to period 2?

5. (20%) A consumer lives for 2 periods and earns $Y_1 = 10\$$, in period 1, and in period 2 he or she earns $Y_2^a = 10\$$ with probability 1/2 (state a) and $Y_2^b = 30\$$ with probability 1/2 (state b). The consumer starts with 0 assets and maximizes

$$U(C_1) + \frac{1}{1.10} E_1 U(C_2) ,$$

where

$$U(C) = 100C - \frac{1}{2}C^2 .$$

Now assume that a stock (equity) exists but no safe bond. Let the amount of equity bought be S (it can be negative). Assume that the stock has a (net) rate of return of 10% if state a occurs [meaning that agent gets back the principal plus 10%] and 20% if state b occurs.

a) Find S (and the implied consumption plan).

b) Explain intuitively why S is negative or positive (there are several motives for going long or short in the stock, and they may work in the same and in opposite directions, which you are asked to spell out).

6. (15%) Explain, in detail, how one could test the CAPM. (There is more than one correct answer.)