ECONOMICS 7344, Spring 2012
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Midterm Exam 2, April 2—5 questions. All sub-questions carry equal weight except where otherwise indicated.

1. (30%) Assume that an agent lives for 2 periods in an economy with perfect Arrow-Debreu markets and no storage. Assume that the endowment of the agent is $y_0 = 2$ in period 1 his or her endowment is $y_1^g = 3$ in the "good state" g. In the "bad state" b the endowment of the agent is $y_1^b = 1$. Assume that the good state happens with probability 1/2. Assume the price of the Arrow security which pays out in the good state is $\frac{10}{33}$ and the price of the Arrow security which pays out in the bad state is $\frac{20}{33}$.
Assume each agent maximizes a utility function

$$\log(C_0) \; + \; E_0 \log(C_1) \; .$$

i) Find the safe rate of interest r.
ii) Find the level of consumption of the agent in periods 0 and 1 and both states of the world.
iii) Interpret the relation between consumption in the good and bad state (why is one higher than the other) and between consumption in the initial and the second period. (You should be able to do that even if you could not numerically solve for them in the previous question.)
Now assume that the agent does not have access to Arrow-Debreu securities but can trade in a risk-less bond.
iv) Write down one equation in one unknown in the amount (B) of the bond purchased.
v) Would B be positive or negative?

2. (15%) Consider the CAPM-model.
a) Let X be an asset whose payout PO_X is X with probability 0.5 and 0 with probability 0.5. Assume that the return r_X has a covariance with the market return (measured in percent) of 10 and that the variance of the market return is 5 (measured in percent). The expected rate of return of the market is 5 percent. Assume that the safe rate of interest is 2 percent. Finally assume the price of the asset is 10.
a) What is the expected return ($\mathrm{E}r_X$) to an investment in X?
b) Find X.

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3. Assume that an agent lives for 2 periods in an economy with perfect Arrow-Debreu markets and no storage. In the second period there are two states "A" and "B."

(5%) Define "actuarially fair prices" for the two Arrow securities. (Make sure to define all variables clearly).

(15%) Prove that prices are actuarially fair if an agent consumes the same amount in states A and B. (Strictly speaking only one of the states will occur, so this is about the agent's plan in period 1). You have to start by deriving certain Euler equations.

4. (25%) Consider two assets. Assume the CAPM holds. Asset A has pay-out PO_A which has a correlation of 0.2 with the market return while asset B has pay-out PO_B which has a correlation of 0.5 with the market return. If the standard deviation of the return to asset A is 2 times the standard deviation of the return to asset B, which asset will have the highest expected rate of return?

5. (10%) Consider the CAPM-model. Assume the safe rate of interest is 3%, the mean return to the market portfolio is 6% and the variance of the (percent) return to the market portfolio is 2. Now consider an assets S with a payout which is normally distributed with mean 200 and variance 10 (again in percentage terms). Assume the covariance of the payout to asset S with the market return is 2.
What would be the price of asset S?