

Midterm Exam 2 — 5 questions with a total of 100 points. All sub-questions carry equal weight.

1. (20%) Consider the AR model

$$a(L)y_t = 37 + u_t ,$$

where u_t is iid white noise with variance 3. $a(L)$ is the lag polynomial $1 - .5 * L$. Consider the model

$$a(L)b(L)z_t = 10 + e_t ,$$

where e_t is white noise with variance 1 and $b(L)$ is the lag polynomial $1 + .5 * L$.

- a) Is the AR-model for z_t stable?

Assume that Hall's PIH-model holds with income described by the z_t model and assume the rate of interest is 10%. Assume the level of consumption C_0 in year 0 equals 10.

- b) Assume that $e_1 = 1$. Find C_1 .

2. (15%) Consider the CAPM-model. Assume the safe rate of interest is 0, the mean return to the market portfolio is 0.10 and the variance of the return to the market portfolio is 0.02. Now consider assets D and E. For these we know the distribution of the pay-outs. For D the payout is normally distributed with mean 100 and variance 10, while for E the payout is normally distributed with mean 200 and variance 60. Assume the covariance of the payout to asset D with the market return is 1 while the covariance of payout to asset E with the market return is 2.

What would be the prices and the expected returns of assets D and E?

3. (20%) Derive the Consumption-CAPM relation.

4. (15%) A consumer lives for 3 periods and earns 1\$, 2\$, and X\$ in period 1, 2, and 3; respectively, where X is a normally distributed variable with mean 10 and variance 9. The consumer maximizes

$$U(C_1) + E_1 U(C_2) + E_1 U(C_3) ,$$

where

$$U(C) = C - .01 C^2 .$$

The consumer can freely borrow and lend at a known interest rate of 10 percent.

a) Find the consumer's period 1 consumption and his/her expected consumption in period 2 and period 3.

5. (30%) A consumer lives for 2 periods and earns $Y_1 = 20\$$ in period 1, and in period 2 he or she earns $Y_2^a = 20\$$ with probability 1/2 (state a) and $Y_2^b = 30\$$ with probability 1/2 (state b). The consumer starts with 0 assets and maximizes

$$U(C_1) + \frac{1}{1.10} E_1 U(C_2) ,$$

where

$$U(C) = 100C - \frac{1}{2} C^2 .$$

Assume that the safe rate of interest is 5 percent.

a. (10%) Let B denote the amount lent in period 1 (or, equivalently, the amount of a safe bond bought). Assuming that the agent doesn't have access to any other assets, find B and the consumption in each period (for period 2, that means the consumption plan listing consumption in state a and state b .)

In the second part of the question, assume that the rate of interest on the bond (lending) is 0 percent and the consumer maximizes

$$U(C_1) + E_1 U(C_2) .$$

(These changes is just to simplify calculations.)

b. (20%) Now assume that a stock (equity) exists besides the safe bond. Let the amount of equity bought be S (it can be negative). Assume that the stock has a (net) rate of return of 0% if state a occurs [meaning that agent gets back the principal] and 100% if state b occurs. Find B and S (and the implied consumption plan). (Note: the question is set up with extreme values to make the algebra easier, so the solution may also be "extreme." Also note, that for the PIH negative values of consumption are valid. If you are running out of time, most points will be accrued when you write down the equations that determines the answer.)