

**Midterm Exam 2, April 7—4 questions. All sub-questions carry equal weight except where otherwise indicated.**

1. (40%) Assume that  $y_t$  follows the stationary AR(1) process

$$y_t = 200 + 0.2y_{t-1} + u_t$$

where  $u_t$  is white noise with variance 2.

- a) (5%) Find the mean and variance of  $y_t$ .

Now assume that the PIH holds such that  $\Delta C_t = \alpha u_t$ . Assume the rate of interest is 10%.

- b) (10%) Find the value of  $\alpha$  (this should be a number).

- c) (5%) What is the variance of consumption growth?

Now you are told that  $y_2 = 210, y_1 = 200$  and  $y_0 = 200$ . (This holds for the remaining questions.)

- d) (5%) What is  $\Delta C_2$  ?

- e) (10%) Assume that the consumer has assets  $A_2 = 0$  at the beginning of period 2. What is  $C_2$ ?

- f) (5%) What is the conditional expectation  $E\{C_3|y_2, y_1, y_0\}$ ?

2. (17%) Derive the consumption CAPM.

3. (18%) A consumer lives for 3 periods and earns 200\$,  $X$ \$, and 200\$ in period 1, 2, and 3 respectively.  $X$  is a normally distributed random variable with mean  $EX = 200$  and variance  $\text{Var}X = 5$ . The consumer has an exponential utility function  $U(C) = -e^{-\theta C}$  and is—in period 1—allowed to freely borrow and lend at an interest rate of 0 (for simplicity). Any borrowing or lending has to be paid back in period 2. For simplicity also assume that the consumer has a 0 rate of time preference ( $\beta = 1$ ). The consumer is not allowed to borrow or lend in period 2 (meaning from period 2 to period 3). Find  $C_1, C_2$  and  $C_3$ .

PLEASE TURN OVER

4. (25%) Consider the CAPM. Assume that there are two states of the economy next year, “A” and “B”; state A happens with probability 0.2 and state B with probability 0.8. Assume the return to the market portfolio is 10% in both states. Now consider assets 1 and 2. For these we know the pay-outs. For asset 1 the payout is 20 in state A and 120 in state B, while for asset 2 the payout is 200 in state A and 5 in state B. Use the CAPM as it was derived in the handout. The safe rate of return is 1%. (Note: it is not really feasible for the market return to have zero variance and a higher rate of return than the safe asset—we make this assumption for simplicity.)
- a) What would be the rate of returns of assets 1 and 2?

Now assume that in state A the return to the market portfolio is 10% and in state B the return to the market portfolio is 0%.

- b) Find the rate of returns to asset 1 and asset 2 under this assumption.
- c) Explain in words why one of the assets (which one) is worth relatively more in this situation compared to the situation where the market return has zero variance.