

**Midterm Exam 2—5 questions. All sub-questions carry equal weight.**

1. (15%) Assume that you are an empirical economist and that Hall's PIH-model holds. Assume that the rate of interest is 10% and that labor-income follows the ARMA-process

$$\Delta y_t = 50 - a\Delta y_{t-1} + u_t - bu_{t-1} ,$$

where  $u_t$  is white noise.

Assume that you figure out that

$$\Delta C_t = \Delta Y_t .$$

What can you conclude about the values of  $a$  and  $b$ ?

2. (15%) Assume that the sale of potatoes follows the process

$$y_t = 50 - .5\Delta y_{t-1} + e_t ,$$

where  $e_t$  is white noise. Assume  $e_t = 100$  for a given period  $t$ .

Find  $E_t y_{t+k} - E_{t-1} y_{t+k}$  for  $k = 0, 1, 2, 3$ .

3. (25%) A consumer has utility function

$$U(c_t) + \frac{1}{1+\rho} EU(c_{t+1}) + \left(\frac{1}{1+\rho}\right)^2 EU(c_{t+2}) + \dots$$

where the time horizon may be finite or infinite (if the time horizon is infinite, it is implicit that the functions are such that the criterion function remains finite). Assume that the consumer has access to financial assets with random (safe as a special case) net returns  $r_{i,t+1}$ .

Prove the general Euler equation for period  $t$  and state the minimal set of assumptions under which this holds for period  $t$ . (Or to put it differently, what is the necessary conditions for which the relation holds involving period  $t$  and  $t + 1$ . You will get partial points for stating sufficient conditions; i.e., a set of conditions for which the relation holds.)

**Please turn over**

4. (30%) A consumer lives for 3 periods and earns 1\$, 2\$, and 3\$ in period 1, 2, and 3, respectively. The consumer maximizes

$$U(C_1) + E_1U(C_2) + E_1U(C_3) ,$$

where

$$U(C) = -\frac{1}{2}C^{-2} .$$

The consumer can freely borrow and lend at a known interest rate. The rate of interest is 10 percent.

a) Find the growth rate of consumption from period 1 to period 2.

Assume instead that

$$U(C) = -\frac{1}{5}C^{-5} .$$

b) Find the growth rate of consumption from period 1 to period 2.

c) Explain intuitively why the rate of growth is different in the two cases (why is it higher in one case than the other?).

5. (15%) A consumer lives for 2 periods and earns  $Y_1 = 10\$$ , in period 1, and in period 2 he or she earns  $Y_2 = 5\$$  with probability  $1/2$  and  $Y_2 = 15\$$  with probability  $1/2$ . The consumer maximizes

$$U(C_1) + E_1U(C_2) ,$$

where

$$U(C) = -\frac{1}{2}C^{-2} .$$

Calculate  $U'(Y_1)$  and  $EU'(Y_2)$  and explain whether the consumer, if given the opportunity, would want to borrow or save in period 1.