

Midterm 2, November 5rd, 2007—4 questions. All sub-questions carry equal weight.

1. (15%) Prove Schwartz inequality.
2. (50%) Consider 2 random variables X and Y . Assume they both are discrete and that X can take the values 0, 1, and 2 while Y can take the values 0,2, and 3. The probabilities for (X,Y) are shown in the following table:

| | | | |
|-----|------|------|------|
| | X=0 | X=1 | x=2 |
| Y=0 | 1/12 | 2/12 | 1/12 |
| Y=2 | 1/12 | 2/12 | 0/12 |
| Y=3 | 2/12 | 2/12 | 1/12 |

- i) Find the marginal probabilities of X and Y .
 - ii) Find the mean of X .
 - iii) Find the variance of X .
 - iv) Are the events $\{X = 1 \text{ or } X=2\}$ and $\{Y = 0\}$ independent events?
 - v) Are the random variables X and Y independent?
 - vi) Find the joint distribution function (CDF) $F(x, y)$.
 - vii) Find the conditional distribution of X given $Y = 0$.
 - viii) Find $Var(X|Y = 0)$.
 - ix) Find the distribution of $2Y^X$.
 - x) If $X + Y = 4$ what is the probability $X = 2$?
3. (15%) Assume $X \sim N(0, 9)$ and $Z \sim N(2, 9)$. Further assume that the covariance between X and Z is 2.
 - i) What is $E(X|Z = 3)$? (State the formula you use and then the number.)
 - ii) What is the conditional variance $Var(X|Z = 2)$?
 - iii) Assume that Y follows the marginal distribution you found in question 2. Assume X is still $N(0, 9)$ and that X is independent of Y . Let $W = X + Y$. Find the conditional distribution of W given Y .
 4. (20%) State and prove the law of iterated expectations.