ECONOMICS 6331, Fall 2006
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Midterm Exam 2 — 5 questions. All sub-questions carry equal weight.

NOTE: We need to be able to follow your calculations, so just giving a number is not considered a full answer (if we really can't follow your reasoning, it is not even a partial answer).

1. (35%) Consider two random variables X and Y. Assume they both are discrete and that X can take the values 1 and 2 while Y can take the values 1,2, and 3. The probabilities for (X,Y) are shown in the following table:

	X=1	X=2
Y=1	0/12	3/12
Y=2	2/12	1/12
Y=3	2/12	4/12

i) Find the marginal distribution of X.
ii) Find the mean and the variance of X.
iii) Are the random variables X and Y independent?
iv) Find the probability $P(\{X > 1\} \bigcap \{Y \leq 2\})$.
v) Find the conditional distribution of X given $Y = 1$.
vi) Find $E(X|Y)$ for $Y = 1$.
vii) Find the distribution of $X^2 Y$.

2. (20%) Let X be a vector random variable with mean μ and variance matrix Σ.
i) Prove that Σ is positive semi-definite.
ii) Prove that the distribution of $A\,X$ is $A\Sigma A'$. (You may use expressions for Σ that we derived in class.)

3. (20%) Let

$$\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 4 \end{pmatrix}$$

be the variance matrix of a vector X where $X = (X_1\,,\,X_2)'$.

i) Find real numbers a, b and c such that $Y_1 = a\,X_1$ and $Y_2 = b\,X_1 + c\,X_2$ are uncorrelated and each have variance 1.
ii) Find $\Sigma^{1/2}$.

PLEASE TURN OVER

4. (15%) Let A be an n-dimensional symmetric matrix such that $A^2 = A$. Prove that if $X_1, ..., X_n$ is a vector of normally distributed random variables that are independent of each other and have variance 1, then $X'AX$ is distributed as χ^2 with degrees of freedom equal to the rank of A.

5. (10%) Let X, Y follow a bivariate normal distribution. Assume that the covariance of X and Y is −2, and the variance of X is 9 and the variance of Y is 4. Further assume that the mean of X is -20 and the mean of Y is 0. If you are told that $Y = 5$, what is the distribution of X?