ECONOMICS 6331, Fall 2005
Bent E. Sørensen

Midterm Exam 2 — 5 questions. All sub-questions carry equal weight.

NOTE: We need to be able to follow your calculations, so just giving a number is not considered a full answer (if we really can't follow your reasoning, it is not even a partial answer).

1. (40%) Consider two random variables X and Y. Assume they both are discrete and that X can take the values 1 and 2 while Y can take the values 1,2, and 3. The probabilities for (X,Y) are shown in the following table:

	X=1	X=2
Y=1	1/12	2/12
Y=2	2/12	1/12
Y=3	2/12	4/12

i) Find the marginal distribution of X.
ii) Find the mean and the variance of X.
iii) Are the events $X = 1$ and $Y = 1$ independent events?
iv) Are the random variables X and Y independent?
v) Find the probability $P(\{X > 1\} \bigcup \{Y \leq 2\})$.
vi) Find the conditional distribution of X given $Y = 2$.
vii) Find $E(X|Y)$ for all Y.
viii) Find $E(X)$ *using your answer to question vii)*. (If you don't use that answer, you will not get points. If you couldn't answer vii) you can make up the answer to that question for use in this one.)
ix) Find $Var(X|Y = 2)$.
x) Find the marginal distribution of Y.

2. (15%) Using the probability distribution from question 1, derive the distribution of $X + Y$.

3. (15%) For the continuous case, derive the formula (convolution formula) for the distribution of $X + Y$.

4. (10%) Assume $X \sim N(1, 1)$, $Y \sim N(2, 9)$, and the covariance between X and Y is 1.
i) What is $E(X|Y = 5)$? (State the formula you use and then the number.)
ii) What is $Var(X|Y = 3)$? (Again, I need to see the formula.)

5. (20%) Assume that X and Y follow a bivariate Normal distribution with non-zero correlation ρ. Denote the mean, variance of X and Y by μ_X, σ_X^2 and μ_Y, σ_Y^2, respectively.
a) State the joint density for X, Y. [PLEASE TURN OVER]

b) Starting from the expression for the joint density, demonstrate that ρ is the correlation coefficient. You may use the formulas for the marginal distributions, and for the conditional distributions without proving them.