

**Midterm Exam 2 — 5 questions. All sub-questions carry equal weight.**

NOTE: We need to be able to follow your calculations, so just giving a number is not considered a full answer (if we really can't follow your reasoning, it is not even a partial answer).

1. (18%) Assume  $X$  and  $Y$  are independent standard exponentially distributed random variables. Derive the density of  $X + Y$ .

2. (18%) Let

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

If  $X_1$  and  $X_2$  are independent, standard  $N(0,1)$ , normally distributed random variables, what is the distribution of  $Y = AX$ , where  $X = (X_1, X_2)'$ .

3. (18%) Assume that  $Y$  is binomially distributed with  $n = 2$  and  $p = 0.4$ . If  $E(X|Y) = Y^2$ , what is  $E(X)$ ?

4. (20%) If  $X$  is an  $n$ -dimensional vector distributed as  $N(\mu, \Sigma)$ , where  $\Sigma$  has full rank, explain in detail why  $(X' - \mu') \Sigma^{-1} (X - \mu)$  is  $\chi^2(n)$  distributed.

5. (26%) Assume that  $X$  and  $Y$  follow a bivariate Normal distribution with non-zero correlation  $\rho$ . Denote the mean, variance of  $X$  and  $Y$  by  $\mu_X, \sigma_X^2$  and  $\mu_Y, \sigma_Y^2$ , respectively.

a) State the joint density for  $X, Y$ .

b) Derive the conditional distribution of  $Y$  given  $X$ .