ECONOMICS 6331, Fall 2004
Bent E. Sørensen
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Midterm Exam 2 — 5 questions. All sub-questions carry equal weight.

NOTE: We need to be able to follow your calculations, so just giving a number is not considered a full answer (if we really can't follow your reasoning, it is not even a partial answer).

1. (18%) Assume X and Y are independent standard exponentially distributed random variables. Derive the density of $X + Y$.

2. (18%) Let

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} .$$

If X_1 and X_2 are independent, standard N(0,1), normally distributed random variables, what is the distribution of $Y = A\,X$, where $X = (X_1\,,\, X_2)'$.

3. (18%) Assume that Y is binomially distributed with $n = 2$ and $p = 0.4$. If $E(X|Y) = Y^2$, what is $E(X)$?

4. (20%) If X is an n-dimensional vector distributed as $N(\mu, \Sigma)$, where Σ has full rank, explain in detail why $(X' - \mu')\,\Sigma^{-1}\,(X - \mu)$ is $\chi^2(n)$ distributed.

5. (26%) Assume that X and Y follow a bivariate Normal distribution with non-zero correlation ρ. Denote the mean, variance of X and Y by μ_X, σ_X^2 and μ_Y, σ_Y^2, respectively.
a) State the joint density for X, Y.
b) Derive the conditional distribution of Y given X.