ECONOMICS 7344, Spring 2015
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Midterm Exam 1, February 23 — 4 questions. All sub-questions carry equal weight.

1. (10%) a) Let

$$x_t \ = \alpha_0 \ + \ 5 * u_t \ + \ u_{t-1} \ ,$$

where u_t is white noise.
Find the autocovariances for x_t in terms of σ_u^2 (the variance of u_t).

b) Given the stationary AR(1) process

$$x_t \ = \ 3 \ + \ .5 * x_{t-1} \ + \ u_t$$

where $Eu_t^2 = 3$. Find the variance of x_t, and the first 3 autocovariances and autocorrelations.

2. (20%) A consumer lives for 2 periods and earns $Y_1 = 20\$$, in period 1, and in period 2 he or she earns $Y_2^a = 10\$$ with probability 1/2 (state a) and $Y_2^b = 30\$$ with probability 1/2 (state b). The consumer starts with 0 assets and maximizes

$$U(C_1) \ + \ E_1 U(C_2) \ ,$$

where

$$U(C) = C - b/2\, C^2 \ .$$

Assume that the safe rate of interest is 0 percent.

a) (5%) Let B denote the amount lent in period 1 (or, equivalently, the amount of a safe bond bought). Assuming that the agent doesn't have access to any other assets, find B and the consumption in each period (for period 2, that means the consumption plan listing consumption in state a and state b.)

b) (15%) Now assume that a stock (equity) exists besides the safe bond. Let the amount of equity bought be S (it can be negative). Assume that the stock has a (net) rate of return of –20% (negative 20%) if state a occurs [meaning that agent gets back the principal] and 60% if state b occurs. Find B and S (and the implied consumption plan).

3. (50%—10% for each subquestion) Assume that income follows the AR(1) process

$$y_t = 2 + 0.8y_{t-1} + e_t \quad (*)$$

where e_t is white noise with variance 3.
a) Is this time-series process stable?

b) Assume that y_0 is a random variable. For what values of the mean $E(y_0)$ and the variance $\text{var}(y_0)$ will the time series y_t; $t = 0, 1, 2, ...$ be stationary?

c) What is $E_1 y_3$ if $y_1 = 5$ and $y_0 = 2$?

d) Write the infinite Moving Average model that is equivalent to the AR(1) model (*) [assuming that the process now is defined for any integer value of t]. (Half the points are from getting the correct mean term.)

e) Consider the PIH model. Assume that the rate of interest is 10 percent and that the consumer have assets $A_t = 110$.

f) Find the *level* of consumption in period t, under the assumption that $y_t = 4$. (Hint: You will need to find $E_t y_{t+k}$.)

g) If $y_{t-1} = 2$ what is $c_t - c_{t-1}$?

4. (20%) Assume that income follows the stationary AR(2) process

$$y_t = 4 + 0.2\, y_{t-1} + 0.5\, y_{t-2} + e_t \quad (*)$$

where e_t is white noise with variance 3.
a) What is the expected value Ey_t.

b) Find the variance of y_t.

c) Assume that you are told that $y_0 = 10$, $y_{-1} = 5$, and $y_{-2} = 5$. Find the conditional expectation $E(y_2|y_0, y_{-1}, y_{-2}, ...)$.