**ECONOMICS 7344, Spring 2014**
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**Midterm Exam 1, February 24—5 questions. All sub-questions carry equal weight.**

1. (15%) Assume that an economy is made up of agents who believe that they have can set the relative price of their production. Each agent has a utility function

$$U(C, L) \ = \ C - \delta L^{\gamma} \ ,$$

where $C$ is consumption, $\delta$ is a positive parameter, $\gamma > 1$, and $L$ is labor supply. Assume that agent $i$ supplies output $Q_i$ produced by the production technology $Q = L$. The agent determines his or her output believing the relative price $P_i/P$,

$$\frac{P_i}{P} = (\frac{Y_i}{Y})^{\frac{-1}{\eta}} \ .$$

where $P$ is the aggregate price index (assume there a many agents so a change in $P_i$ doesn't change $P$), $\eta > 1$, and $Y$ is aggregate (average) output.

a) Find the optimal output as seen by the agent (where the agent takes $P$ as given and believes $C = L_i * (P_i/P)$, where the relative price is determined from relative output as in the formula above). This will be a function of $Y$.
b) Find the equilibrium level of output in the economy (impose that $Y_i = Y$).
c) What is the optimal amount of output in this economy as a social planner would set it?

2. (25%) Consider the AR model

$$y_t \ = \ 10 \ + \ 0.2y_{t-1} \ + \ u_t \ ,$$

where $u_t$ is iid white noise with variance 3.

a) Is this model stable? (Explain our answer, if you state "yes" or "no" with no more elaboration, you get 0 points even if your answer is correct.)
b) Assume $y_0$ is a fixed constant, $y_0 = 5$. Calculate the variance of $y_t$ for all $t$.
c) What is $E_0 y_1$? What is $E_0 y_2$?
$a(L)$ is the lag polynomial $1 + .5 * L$ and $b(L) = 1 + .3 * L$, define $c(L) = a(L) * b(L)$.

d) Find $c(L)$.
e) Define the model $c(L)z_t = u_t$, where $u_t$ is white noise. Is this model stable? (Explain your answer.)

**Please turn over.**

3. (15%) Assume that an economy is inhabited by identical agents (or a representative agent) with utility function of the form

$$\frac{1}{1-\rho}C_1^{1-\rho} + \frac{1}{1.01}\frac{1}{1-\rho}C_2^{1-\rho}$$

a) If the (non-stochastic) growth rate of consumption is 4 percent from period 1 to period 2 and $\rho = 2$ what is the safe rate of interest (the rate of interest from period 1 to period 2)? (Use the approximation $(\log(1+x) = x)$. Explain intuitively why the rate of interest is higher or lower than the discount rate.
b) If instead $\rho = 4$ what is the safe rate of interest? Explain intuitively why the rate of interest is higher or lower than in the answer to part a. (You will only get points for the explanation because the derivation is similar to that of part a.)

4. (15%) Explain what is meant by excess smoothness of consumption. You need to be very specific about each component of your explanation.

5. (30%) Assume that an agent's wage income follows the AR(1) process

$$y_t = 300 + 0.5y_{t-1} + e_t \quad (*)$$

where $e_t$ is white noise with variance 3.

Assume the agent's wage was 100$ in period 1.
a) What is the agents expected wages in period $t$ (for any $t > 1$)?
b) If the rate of interest is 10 percent and the PIH holds, what is the agent's level of consumption in period 1 assuming that his or her assets at the beginning of period 1 was 1000$.
c) Under the assumption of part b), if the agent's wage also was 100$ in period 0, what would be the value of $\Delta C_1$?