ECONOMICS 7344, Spring 2013
Bent E. Sørensen
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Midterm Exam 1 — 6 questions. All sub-questions carry equal weight.

1. (15%) Let y_t be a stationary time series, and $\gamma(k)$ the k-th order autocorrelation.
a) Prove that $\gamma(k) = \gamma(-k)$.
b) If $a(L)$ is the lag polynomial $1 + .4 * L$, find the inverse of $a(L)$. (Show, at least, the first 3 terms.)
c) If $b(L) = 1 + L - 2 * L^2$ find $a(L) * b(L)$.

2. (15%) Assume that output is determined by the "Keynesian Cross"

$$C_t + I_t + G_t = Y_t, \quad t = 1, 2.$$

where $I_t = 0.5 * Y_t$ for $0 < \alpha < 1$, $G_1 = G_2 = 10$. Finally, assume that C is determined by the PIH with Y as the income variable (and no initial assets) with an interest rate of 0.
a) Assume $C_1 = 5$ find Y_2. What is Y_2 if $C_1 = 50$?

b) What does this tell you about the compatibility of the PIH and "Keynesian Cross" [hint, how many equations do you have to determine C and Y, in light of the results from part a)?]

3. (20%) a) Explain what is meant by "menu costs" and why menu costs may lead to price stickiness. (A brief explanation of the main point is sufficient.)
b) Explain (using a figure is best) how small menu cost may have a large impact on welfare.

4. (10%) Assume that an economy is inhabited by identical agents (or a representative agent) with logarithmic utility functions.
If the (non-stochastic) growth rate of consumption is 4 percent from period 1 to period 2 and agents have a discount factor of 0.99 (you can approximate the discount rate by 0.01), what is the safe rate of interest (the rate of interest from period 1 to period 2)?

5. (20%) A consumer lives for 2 periods and earns $Y_1 = 10\$$, in period 1, and in period 2 he or she earns $Y_2^a = 10\$$ with probability 1/2 (state a) and $Y_2^b = 30\$$ with probability 1/2 (state b). The consumer starts with 0 assets and maximizes

$$U(C_1) + E_1 U(C_2) ,$$

where

$$U(C) = \log(C) .$$

Assume that the safe rate of interest is 10 percent.

a.) Let B denote the amount lent in period 1 (or, equivalently, the amount of a safe bond bought). Assuming that the agent doesn't have access to any other assets, find B and the consumption in each period (for period 2, that means the consumption plan listing consumption in state a and state b.)

b.) Now assume that a stock (equity) exists besides the safe bond. Let the amount of equity bought be S (it can be negative). Assume that the stock has a (net) rate of return of 10% if state a occurs [meaning that agent gets back the principal] and 20% if state b occurs. Find B and S (and the implied consumption plan).
Derive (but don't solve because it gets messy) two equations in two unknowns that determines B, and S.

6. (20%) Consider an agent with von Neumann-Morgenstern utility index

$$U(C_0) + \beta E_0 U(C_1) + \beta^2 E_0 U(C_2) + ...$$

Derive the Euler equation for an asset (with stochastic return) that can be purchased in period 0 with a payout in period 2 (in way similar to the way I derived the Euler equation for a regular 1-period investment). Make clear what assumptions are needed.