

Midterm Exam 1, March 5—4 questions. All sub-questions carry equal weight except where otherwise indicated.

1. (30%) Assume that y_t follows the AR(2) process

$$y_t = 100 + 0.5y_{t-1} + 0.3y_{t-2} + e_t \quad (*)$$

where e_t is white noise with variance 2.

- a) (8%) Find the mean and variance of y_t assuming that y_t is stationary.

Now assume that you are told that $y_1 = 200$, $y_0 = 200$, and $y_{-1} = 100$.

- b) (8%) What is the conditional expectation $E(y_3|y_0, y_1)$?

- c) (14%) Assume that the PIH holds. With the assumption made, what is ΔC_1 ?

2. (20%) a) Explain what is meant by excess sensitivity of consumption and how this is typically tested for.

- b) Explain in detail what macroeconomists mean by excess smoothness of consumption.

3. (25%) Assume a consumer maximizes the utility of consumption of N goods C_1, \dots, C_N using the utility index $U(C_1, \dots, C_N) = (\sum_{i=1}^N C_i^{\frac{\eta-1}{\eta}})^{\frac{\eta}{\eta-1}}$. The price of good i is P_i and the consumer faces the budget constraint $\sum_{i=1}^N P_i C_i = Y$ where Y is the consumer's exogenous income.

One can find that $C = Y/P$ for a price index P , where $C = (\sum_{i=1}^N C_i^{\frac{\eta-1}{\eta}})^{\frac{\eta}{\eta-1}}$. Derive P and verify that $C = Y/P$ (assume that each quantity is so small it doesn't affect P). Also find the optimal C_i in terms of P_i , P , and Y . (You will get 5 points for just stating the right formula for P).

PLEASE TURN OVER

4. (25%) Assume that an agent optimizes expected utility

$$\sum_{t=0}^{\infty} \left(\frac{1}{1.1}\right)^t U(C_t) ,$$

where $U(C) = 100C - .5C^2$ (assume the numbers are such that marginal utility is always positive). Further assume that the agents income follows the process

$$Y_t = 5 + .2Y_{t-1} + u_t ,$$

and that the agent has zero assets at period 0. Assume that $Y_0 = 10$ and the rate of interest is 5 percent.

Find C_0 and comment on the magnitude compared to the constant (5) in income and the initial level of income (10).