

**Midterm Exam 1, March 1—3 questions. All sub-questions carry equal weight except where otherwise indicated.**

1. (30%) Assume that  $y_t$  follows the AR(2) process

$$y_t = 200 + 0.5y_{t-1} + 0.3y_{t-2} + e_t \quad (*)$$

where  $e_t$  is white noise with variance 4.

- a) (8%) Is this process stable? (You need to show why).  
b) (8%) Find the mean and variance of  $y_t$  assuming that  $y_t$  is stationary.

Assume that you are told that  $y_1 = 500$  and  $y_0 = 400$ . (But for question d) you should still think of these as realizations from the stationary distribution.)

- c) (6%) What is the conditional expectation  $E(y_3|y_0, y_1)$ ?  
d) (8%) Prove that  $E(y_1|y_0) = E(y_0|y_1)$ ? (Hint: One way is to make the assumption that  $y_1$  and  $y_0$  are jointly normally distributed and drawn from the stationary distribution and use what you know from 6331.)

2. (35%) The Fischer model.

- a) (20%) Derive the formulas for prices as a function of  $m_t$  and lagged expectations of  $m_t$  in the Fischer model. (Start from the assumption that the desired price level in log terms is  $p_t^* = 0.5m_t + 0.5p_t$ .)

- b) (15%) Assume that monetary policy is described by the AR(2) process

$$m_t = 10 + 0.5m_{t-1} + 0.3m_{t-2} + e_t \quad .$$

Also assume you observe  $m_t = 10$ ,  $m_{t-1} = 20$ , and  $m_{t-2} = 40$ .  
What is the predicted level of  $y_t$  and  $p_t$  in the Fischer model?

PLEASE TURN OVER

3. (35%) Assume that a representative agent has a utility function

$$U(C, L) = C - \delta \exp(L) ,$$

where  $\delta$  is a positive parameter. Assume that agent  $i$  supplies output  $Q_i$  produced by the production technology  $Q = L$ . The agent sets the relative price  $P_i/P$ , where  $P$  is the aggregate price index (assume there are many agents so a change in  $P_i$  doesn't change  $P$ ) and faces a demand function

$$Q_i = Y \left( \frac{P_i}{P} \right)^{-\eta} .$$

The agent supplies labor  $L_i$  to the market at the equilibrium wage rate  $W$  and hires labor in the amount of  $Q_i$ .

- a) Find the optimal relative price  $\frac{P_i}{P}$  (where the agent takes  $P$  as given).
- b) Find the agent's labor supply as a function of the real wage.
- c) Find the equilibrium level of output in the economy.
- d) Does the equilibrium level of output increase or decrease with the parameters  $\delta$  and  $\eta$ ? Explain the intuitive logic underlying your answer.