ECONOMICS 7344, Spring 2008
Bent E. Sørensen
March 3, 2008

Midterm Exam 1, March 3—4 questions. All sub-questions carry equal weight except where otherwise indicated.

1. (20%) Assume that y_t follows the AR(2) process

$$y_t = 200 + 0.5y_{t-1} + 0.1y_{t-2} + e_t \quad (*)$$

where e_t is white noise with variance 2.

a) (8%) Find the mean and variance of y_t assuming that y_t is stationary.

Now assume that you are told that $y_1 = 200$ and $y_0 = 200$.
b) (4%) What is the conditional expectation $E(y_2|y_0, y_1)$?
c) (4%) What is the conditional expectation $E(y_3|y_0, y_1)$?
d) (4%) What is the conditional expectation $E(y_1|y_0)$? (Hint: Make the assumption that y_1 and y_0 are jointly normally distributed and use what you know from 6331.)

2. (30%) Assume output in an economy is determined by the equilibrium condition that aggregate demand E is equal to total output Y. Assume that $E = C + I + G$ where consumption $C = 2 + .6 * (Y - T)$ (T is net taxes), investment is exogenous at 1, and government consumption is exogenous at 2. T equals 2. Assume that inflation and expected inflation is 0 and that $P = 1$. Money supply is exogenous at 10, and the demand for money is $P * L(Y, r)$ where $L(Y, r) = Y - 0.1 * r$. (The numbers are not chosen to give reasonable values for the solution, so don't worry about "crazy" interest rates etc.)
a) Derive the IS curve (meaning give the equation with the actual intercept and slope implied by the numbers given).
b) Derive the MP-curve (in Romer's notation) (equivalent to the LM curve here where inflation is 0). Again, we need the actual equation with intercept and slope.
c) Solve the model for the equilibrium level of output and interest rate.
d) What happens if M doubles to 20? (Find the new level of output and interest rate.)
e) Assume that government consumption and net taxes both doubles to 4. What is the effect on output?

PLEASE TURN OVER

3. (20%) If one estimates the consumption function

$$c_{it} = \beta_0 + \beta_1 y_{it} + u_{it}$$

for a sample of individuals where c_{it} is non-durable consumption and y_{it} is current income of individual i at period t, one gets a very low (positive, but closer to 0 than to 1) estimate of β_1.
a) Explain why this might be (follow Friedman's explanation).
It was also observed that when one estimates the model

$$c_t = \alpha_0 + \alpha_1 y_t + w_t \ ,$$

using aggregate data, one gets a larger estimate of α_1 than one gets using individual data.
b) What was Friedman's explanation for this?

4. (30%) Consider an economy with a large number of agents where the utility of agent i is determined by a utility function

$$U(C_i, L_i) \ = \ E \log C_i - \alpha L_i \ ,$$

where L_i is labor supplied, C_i is agent i's consumption (a basket of goods in fixed proportions) and α is a positive parameter (E is the expectations operator). Assume that agent i supplies output Q_i produced by the production technology $Q = L$. The agent is a price taker and the price of the single good agent i produces is denoted P_i. The aggregate price index (price of consumption) is $P = 1$ so $C_i = P_i * Q_i$. Assume there are many goods so a change in P_i doesn't change P. Agent i faces a demand function

$$Q_i = Y \, P_i^{-1} Z_i,$$

where Y is aggregate output and Z_i is log-normally distributed with mean $e^{\sigma_z^2/2}$, where $\sigma_z^2 = 2$ is the variance of $\log(Z_i)$. Assume that the Z_i random variables are independent of each other and independent of Y. Assume that the agent has to decide on his labor supply *before* he or she knows Z_i (otherwise there will no uncertainty at all).

a) (15%) Find the equilibrium level of output in the economy. (You need to solve the model. Hint: If you consider the relation between normal and log-normal random variables, you can figure out what is the distribution of Z_i^{-1}.)
b) (5%) Explain intuitively why output goes up/goes down/stays the same, when α increases. You can get full points if you explain what must happen even if you couldn't solve part a).

Now assume instead that

$$U(C_i, L_i) \ = \ E\{ \, C_i - \kappa \frac{1}{2} C_i^2 \} \ - \ \alpha L_i \ .$$

c) (10%) Find the level of output using this utility function (assume that the magnitudes of κ and α are such that a positive solution exists).