ECONOMICS 7344, Spring 2006
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February 25, 2007

Midterm Exam 1, February 22 — 4 questions. All sub-questions carry equal weight except where indicated.

1. (15%) Assume that income follows the AR(1) process

$$y_t = 20 + 0.2\, y_{t-1} + e_t \quad (*)$$

where e_t is white noise with variance 3 and y_0 equals 4.
a) Is this time-series process stable? Is it stationary?
b) What are the expected values of y_1 and y_2?
c) Assume now that the process is stationary and defined for all integer t. (In other words, from now on it is NOT the case that y_0 is a given number.) Write the infinite Moving Average model that is equivalent to the AR(1) model (*). (Half the points are from getting the correct mean term.)

2. (15%) Assume that income follows the stationary AR(2) process

$$y_t = 4 + 0.2\, y_{t-1} + 0.5\, y_{t-2} + e_t \quad (*)$$

where e_t is white noise with variance 3.
a) What is the expected value Ey_t.
b) Find the variance of y_t.
c) Assume that you are told that $y_0 = 10$, $y_{-1} = 5$, and $y_{-2} = 5$. Find the conditional expectation $E(y_2|y_0, y_{-1}, y_{-2}, ...)$.

3. (60%) Assume an economy with a large number of agents where the utility of agent i is determined by a utility function

$$U(C_i, L_i) \;=\; C_i - L_i^{\delta} \;,$$

where L_i is labor supplied, C_i is agent i's consumption (a basket of goods in fixed proportions) and $\delta > 1$ is a positive parameter. Assume that agent i supplies output Q_i produced by the production technology $Q = L$. The agent is a price taker and the price of the single good agent i produces is denoted P_i. The aggregate price index (paid for consumption) is P and $C_i = P_i * Q_i/P$. Assume there are many goods so a change in P_i doesn't change P. Agent i faces a demand function

$$Q_i = Y \, (\frac{P_i}{P})^{-1} E_i \;,$$

where Y is aggregate output and E_i is log-normally distributed with mean $e^{\sigma_z^2/2}$, where σ_z^2 is the variance of $\log(E_i)$. Assume that the E_i components are independent of each other.

Further assume that the aggregated demand equation $m = y + p$ holds, where $m = \log(M)$, $p = \log(P)$, and $y = \log(Y)$. Similarly, define y_i, p_i, etc. as the logarithm of the variables defined by capital letters. Assume that y is the average value of y_i and p is the average value of p_i.
The equations given hold in any time period t. Assume that m follows the time-series process

$$m_t = 10 + 0.6\, m_{t-1} + 0.2\, m_{t-2} + \epsilon_t \ ,$$

where ϵ_t is white noise with variance 4.
a) (20%) Find the equilibrium level of output in the economy. (You need to solve the model.)
Now assume that agents don't observe the aggregate price level P and that agents determine their supply using certainty equivalence. (In the current period, they do observe last periods aggregate price level.) Assume that $m_t = 12$, $m_{t-1} = -10$ and $m_{t-2} = -5$.
b) (40%) Find the level of output in period t.
NOTE: To get credit I need to be able to follow your reasoning. So, for example, in part b) you should first derive the formula for output and then plug in the numbers afterwards. If there is some coefficient you cannot derive, the best strategy is to choose some value and then continue, if you get the rest correct conditioned on that value you will get most points.

4. (10%) Derive the AD (aggregate demand) curve following Romer. Start from money demand functions, consumption functions, etc.