

**Midterm Exam 1 — 5 questions. All sub-questions carry equal weight unless otherwise noted.**

NOTE: We need to be able to follow your calculations, so just giving a number is not considered a full answer (if we really can't follow your reasoning, it is not even a partial answer).

1. (15%) Consider a uniform distribution on the set  $[0, 4] \cup [10, 12]$ . Assume a random variable  $X$  follows this distribution.

- a) What is the density function (PDF)?
- b) Find the mean of  $X$ .
- c) Find the variance of  $X$ .

2. (30%) Consider a sample of computers. The probability of observing a Dell is 40% and the probability of observing an HP is 60%.

a) If you select 10 computers at random what is the probability that they are all HP's?

Now assume that 10% of HP's are defect and 20% of Dells are defect.

- b) If you select one computers at random what is the probability that it is defect?
- c) If you select N computers are random (from a sample that we consider infinitely large), what is the probability that they are all defect?
- d) What is the expected number of defect computers if you select N?
- e) If you select one computer and it is defect, what is the probability that it is a Dell?
- f) If the Dell computers have 1 Gigabyte of memory and HP computers have 2 Gigabytes of memory and you select 2 computers at random, what is the expected total amount of memory in the 2 computers.

3. (25%) a) Derive the exponential distribution by taking an appropriate limit (as we did in class).

b) Find (meaning that you have to derive it) the moment generating function for the exponential distribution.

c) Find the mean and variance for the exponential distribution (you can use your answer to part b) or find them directly).

4. (15%) a) Prove Bayes Law?

b) Prove that  $P(A \cup B|C) = P(A|C) + P(B|C) - P(A \cap B|C)$  for any sets A, B, and C.

5. a) (10%) If  $X$  is uniformly distributed on the interval  $[2, 6]$ , find the distribution of  $Y = X^2$ .

b) (5%) Find the mean of  $Y$ .