Midterm Exam 1 — 5 questions. All sub-questions carry equal weight.

NOTE: We need to be able to follow your calculations, so just giving a number is not considered a full answer (if we really can't follow your reasoning, it is not even a partial answer).

1. (25%) Consider a uniform distribution on the closed interval $[-1, 4]$. Assume a random variable X follows this distribution.
a) What is the Cumulative Density Function (CDF)?
b) What is the density function (PDF)?
c) Find the mean of X.
d) Find the variance of X.

2. (25%) Consider a sample of students. The probability of observing a male is 1/3 and the probability of observing a female is 2/3. Now assume the following (made up numbers), namely that the probability that a male is in law school is 20% and the probability that a female is in law school 10%.
a) If you select one person from the population according to these probabilities, what is the probability that you will observe a male student in law school? (Explain how you arrive at you answer)
b) What is the probability that an observed student is a male if you know that the student is in law school?
c) If you select a number of students one after the other. What is the probability that that you have to select X students before you observe a male?

3. (25%) a) Derive the Poisson distribution by taking the appropriate limit of Binomial distributions.
b) Find the moment generating function for the Poisson distribution.
c) Find the mean and variance for the Poisson distribution.

4. (10%) a) What is the formula for the probability of an event A conditional on an event B?
b) Derive the formula for $P(A \bigcup B)$.

5. (15%) If X is exponentially distributed with mean 1, find the distribution of X^2.