

Midterm Exam 1 — 5 questions. All sub-questions carry equal weight.

NOTE: We need to be able to follow your calculations, so just giving a number is not considered a full answer (if we really can't follow your reasoning, it is not even a partial answer).

1. (28%) Consider a uniform distribution on the closed interval $[0, 1]$. Assume a random variable X follows this distribution.
 - a) Find the mean of X .
 - b) Find the distribution of $Y = \log(X)$. (Be specific about all details of the distribution.)
 - c) Find $P(Y < -0.5)$.
 - d) Find $E(Y)$.

2. (20%) Suppose that you consider 3 events: A : You pass the core exam. B : You get an A in statistics. C : The Astros (Houston sports team) wins the World Series. Assume that $P(A) = 1/3$, $P(B) = 1/2$ and $P(C) = 1/5$. Further assume that the event C is independent of both A and B (and all subsets of these). Finally, we assume that $P(A|B) = 2/3$.
 - a) What is the probability that you will pass both the core exam and the statistics exam?
 - b) What is the probability that either the Astros win or you get an A in statistics or you pass the core?
 - c) Assuming that a clairvoyant tells you that you will pass the core. Given that, what will be the probability that you will get an A in statistics?
 - d) What is the probability $P(A \cup C|B)$?

3. (20%)
 - a) Explain (give the precise expression) what is meant by excess kurtosis.
 - b) Does a uniform distribution on $[-5, 5]$ have excess kurtosis?

4. (20%) Derive the Moment Generating Function for a normal distribution with mean μ and variance σ^2 .

5. (12%) Assume that X is binomially distributed with $p = 0.2$ and $n = 2$.
 - a) Find $P(X > 2)$
 - b) Find the probability distribution for $Y = 2 + X^2$.