

Midterm Exam 1 — 6 questions. All sub-questions carry equal weight.

NOTE: We need to be able to follow your calculations, so just giving a number is not considered a full answer (if we really can't follow your reasoning, it is not even a partial answer).

1. (20%) Consider a uniform distribution on the closed interval $[-4, 4]$. Assume a random variable X follows this distribution.
 - a) What is the Cumulative Density Function (CDF)?
 - b) What is the density function (PDF)?
 - c) Find the Moment Generating Function.
 - d) Find the mean of X .
 - e) Find the variance of X .

2. (20%) Suppose we have some observations of Texans and Californians. The probability of observing a Texan is $1/3$ and the probability of observing a Californian is $2/3$. Now assume the following (made up numbers), namely that the probability that a Texan is a republican is 40% (so the probability that he is a democrat is 60%, we assume), and the probability that a Californian is a republican is 50% (so the probability that a Californian is a democrat is also 50%).
 - a) If you select one person from the population according to these probabilities, what is the probability that you will observe a republican from Texas? (Explain how you arrive at your answer)
 - b) In the model described for Californians and Texans, are the events A : {A person is a democrat} and the event B : {A person is from California} independent events? (Explain how you find the answer).
 - c) If you select 5 people randomly from the Texans. What is the expected number of republicans?

3. (20%) Assume that X follows a log-normal distribution with density $\frac{1}{\sigma x \sqrt{2\pi}} e^{-(\ln x)^2 / 2\sigma^2}$ for $x > 0$.
 - a) What is the mean of X ?
 - b) Find $P(X < 1)$.

4. (10%)
 - a) What is the formula for the probability of an event A conditional on an event B ?
 - b) Derive Bayes formula.

5. (10%) If X is exponentially distributed with mean 1, what is the distribution of e^{-X} ?

6. (20%)
 - a) State a version of Chebychev's theorem. (This needs to be precise.)
 - b) Prove Chebychev's theorem. (Make sure each step is clear.)