Midterm Exam 1 — 6 questions. All sub-questions carry equal weight.

NOTE: We need to be able to follow your calculations, so just giving a number is not considered a full answer (if we really can't follow your reasoning, it is not even a partial answer).

1. (20%) Consider a uniform distribution on the closed interval $[-4\,,\,4]$. Assume a random variable X follows this distribution.
a) What is the Cumulative Density Function (CDF)?
b) What is the density function (PDF)?
c) Find the Moment Generating Function.
d) Find the mean of X.
e) Find the variance of X.

2. (20%) Suppose we have some observations of Texans and Californians. The probability of observing a Texan is $1/3$ and the probability of observing a Californian is $2/3$. Now assume the following (made up numbers), namely that the probability that a Texan is a republican is 40% (so the probability that he is a democrat is 60%, we assume), and the probability that a Californian is a republican is 50% (so the probability that a Californian is a democrat is also 50%).
a) If you select one person from the population according to these probabilities, what is the probability that you will observe a republican from Texas? (Explain how you arrive at you answer)
b) In the model described for Californians and Texans, are the events A: {A person is a democrat} and the event B: {A person is from California} independent events? (Explain how you find the answer).
c) If you select 5 people randomly from the Texans. What is the expected number of republicans?

3. (20%) Assume that X follows a log-normal distribution with density $\frac{1}{\sigma x \sqrt{2\pi}} e^{-(\ln x)^2 / 2\sigma^2}$ for $x > 0$.

a) What is the mean of X?
b) Find $P(X < 1)$.

4. (10%) a) What is the formula for the probability of an event A conditional on an event B?
b) Derive Bayes formula.

5. (10%) If X is exponentially distributed with mean 1, what is the distribution of e^{-x}?

6. (20%) a) State a version of Chebychev's theorem. (This needs to be precise.)
b) Prove Chebychev's theorem. (Make sure each step is clear.)