

Midterm Exam 1 — 5 questions. All sub-questions carry equal weight.

1. (30%) Consider a uniform distribution on the closed interval $[1, 4]$. Assume a random variable X follows this distribution.
 - a) What is the Cumulative Density Function (CDF)?
 - b) What is the density function (PDF)?
 - c) Find the Moment Generating Function.
 - d) Find the mean of X .
 - e) Find the variance of X .

2. (24%) A study of college students finds that while 60 percent of college students are male, only 40 percent of college students with an A average are male. In contrast, 15 percent of female students have an A average. Assuming these results are accurate answer the following questions.
 - a) Are “being a male student” and “having an A average” independent? Why?
 - b) What is the probability that a randomly selected student has an A average?
 - c) What is the probability that a randomly selected male student has an A average?

3. (24%) Assume that X follows a standard exponential distribution with density e^{-x} for $x > 0$.
 - a) What is the density function for Y if $Y = 2X$?
 - b) Find $P(X < 1)$.
 - c) Find the 10% upper percentile for X .
 - d) Now assume that you are told that $X < 2$. Given that, what is $P(X < 1)$?

4. (12%)
 - a) Define “excess kurtosis.”
 - b) State Chebychev’s inequality.
 - c) What is the formula for the probability of an event A conditional on an event B ?

5. (10%) For a random variable X , a constant c , and two functions $g()$ and $h()$ prove that
 - a) $E[g(X) + h(X)] = E[g(X)] + E[h(X)]$.
 - b) $E[cg(X)] = cE[g(X)]$.