Homework 3. Due Tuesday March 22. (NOTE: Do as much as you can. We will give full grade for effort. I need to figure out how far you are. I believe you should be able to solve this, but I may be wrong. The sooner you give me feed-back, the better for everybody.)

1. Suppose that in a population of n individuals the **true** relationship between two variables, x and y, is:

$$y_i = \alpha + \beta * x_i + u_i = 10 - 1 * x_i + u_i, \qquad i = 1, \dots, 100$$

where the above relationship satisfies all the assumptions of the linear regression model. Suppose also that you know *a priori* that the variance of $\hat{\beta}$ is equal to 1.

Now consider a test of the hypothesis, $H_0 : \beta = 0$ (and the alternative $H_1 : \beta \neq 0$) in which the hypothesis is rejected if the absolute value of the estimated coefficient, $\hat{\beta}$, is greater than some critical value k. That is, the null hypothesis is rejected if $|\hat{\beta}| > k$, where k is the critical value of the test.
a) What critical value would you choose for this test if you wanted a 5% level of significance?
b) Given this test and a 5% level of significance, what is the probability of making a type II error?
c) By repeating the kind of calculation that you did in b) make a rough sketch of the power function.

2. Let $x_1 = 2,\ x_2 = 0,\ x_3 = 3, x_4 = 2$ and let $y_1 = 2,\ y_2 = 0,\ y_3 = -2,\ y_4 = 4$.
a) Find $\bar{x}$, $\bar{y}$, S_{xx}, S_{yy}, and r_{xy} (put your calculations in a little table so I can see what you do).
b) Plot y against x (just do a little sketch by hand) relate that to the value you found for r_{xy}.
c) Find the estimated regression line and the estimated standard deviation for the slope.
d) Perform a t-test for the significance of the slope. (Show all the steps - so we can see what you did).

3. Matrix exercises.

1. Prove that
$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}^{-1} = \begin{pmatrix} A_{11}^{-1}B & -A_{11}^{-1}A_{12}F \\ -FA_{21}A_{11}^{-1} & F \end{pmatrix} \tag{1}$$
where $B = I + A_{12}FA_{21}A_{11}^{-1}$ and $F = (A_{22} - A_{21}A_{11}^{-1}A_{12})^{-1}$.

2. Let A be a symmetric idempotent matrix. Prove that $\text{tr}A = \rho(A)$.

3. Prove that $\text{tr}A'A = \sum_i \sum_j a_{ij}^2$.

4. Show $\sum_(y_i - \bar{y})^2 = y'M^0y$ with $M^0 = I_N - \frac{1}{N}\iota_N\iota_N'$. ($\iota_N$ is an N-dimensional vector of ones, I_N an N-dimensional identify matrix.)