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ECONOMICS 6331 – Probability and Statistics, Fall 2007

Homework 9. Wednesday November 7, 2007. Due Monday November 12.

1. If

$$\Sigma = \begin{pmatrix} 20 & 10 \\ 10 & 10 \end{pmatrix}$$

and

$$\Sigma^{1/2} = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$$

verify that $\Sigma = \Sigma^{1/2}\Sigma^{1/2'}$. Also find the inverse of $\Sigma^{1/2}$ which we refer to as $\Sigma^{-0.5}$ and Σ^{-1} and verify that that $\Sigma^{-0.5'}\,\Sigma^{-0.5} = \Sigma^{-1}$.

2. Derive the formula for the bivariate Normal distribution from the matrix formula for the multivariate normal. First show that the formula agrees with the formula for the usual univariate Normal if $N = 1$. Then, by inverting the variance matrix etc., verify that the formula for the multivariate Normal is identical to the formula for the bi-variate Normal distribution when $N = 2$. (What I want you to do is to start from the matrix notation and then derive the formula involving variances, correlation coefficient, etc. that doesn't involve matrices or vectors.)

3. Ramanathan, Exercise 5.7, page 118.

4. Let

$$\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 4 \end{pmatrix}$$

be the variance matrix of a vector X where $X = (X_1\,,\, X_2)'$ and $E\,X' = (1, 4)$. Let $a = (2\,,\, 3)'$ and

$$B = \begin{pmatrix} 1 & 4 \\ 8 & 0 \end{pmatrix}.$$

If $Y = a + B\,X$, what is $E\,Y$ and the variance of Y?