Bent E. Sørensen

ECONOMICS 6331 – Probability and Statistics, Fall 2006

Homework 9. Wednesday November 15, 2005. Due Monday November 20.

1. Ramanathan, Exercise 5.22, page 121.

2. Show that if X is t-distributed with q degrees of freedom, then X^2 is $F(1, q)$ distributed.

3. Assume X_n converges in probability to a constant c and Y converges in probability to a constant d. Prove that the random variable $W_n = X_n + Y_n$ converges in probability to $c + d$.

4. Assume that X is an n-dimensional random variable with covariance matrix Σ and Y is an n-dimensional random variable, independent of X with covariance matrix Ω. Show that the covariance matrix for $X + Y$ is $\Sigma + \Omega$. (If you have problems with the general situation, we will give full point is you show it for 2-dimensional case.)

5. a) If X is an $n \times k$ matrix of rank k (with $k \leq n$), verify that $M = X\,(X'X)^{-1}X'$ is an idempotent matrix and that $(I - M)$ is idempotent.
b) Let

$$X = \begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{pmatrix}$$

and find M.
c) What are the ranks of M and $(I - M)$?