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ECONOMICS 6331 – Probability and Statistics, Fall 2008

Homework 8. Wednesday November 19, due Monday November 24.

1. Ramanathan, Exercise 5.22, page 121.

2. a) If X is an $n \times k$ matrix of rank k (with $k \leq n$), verify that $M = X\left(X'X\right)^{-1}X'$ is an idempotent matrix and that $(I - M)$ is idempotent.
b) Let

$$X = \begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 0 \end{pmatrix}$$

and find M.
c) What are the ranks of M and $(I - M)$?

3. (12% of 2004 final) Assume $X \sim \chi^2(9)$.
a) What is $E(X)$?
b) Derive the formula for the variance of a $\chi^2(k)$ (chi-square with k degrees of freedom) random variable.

4. Let

$$\Sigma = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$$

Find $\Sigma^{-1/2}$ using the formula for the conditional mean *or* finding it directly without thinking about random variables.