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ECONOMICS 6331 – Probability and Statistics, Fall 2008

Homework 7. Wednesday November 12, 2008. Due Wednesday November 19.

1. Assume that X and Y follows a bivariate normal distribution.
a) Show that $X - E(X|Y)$ is independent of Y. (Use the law of iterated expectations or just find the covariance.)
b) Find the variance of $X - E(X|Y)$ (hint: This a linear function of X and Y).

c) Now show that $\frac{1}{(1-\rho^2)}[(\frac{X-\mu_X}{\sigma_X})^2 - 2\rho(\frac{X-\mu_X}{\sigma_X})(\frac{Y-\mu_Y}{\sigma_Y}) + (\frac{Y-\mu_Y}{\sigma_Y})^2]$ is distributed as $\chi^2(2)$.

2. (12% of 2003 final) Assume $X \sim N(0,9)$, $Y \sim N(2,9)$, and $Z \sim N(2,16)$. Further assume that the covariance between X and Y is 2, while both X and Y are independent of Z.
i) What is $E(X|Y = 2, Z = 3)$? (State the formula you use and then the number.)
ii) What is the conditional variance $Var(X|Z = 3)$?

3. Assume that X is an n-dimensional random variable with covariance matrix Σ and Y is an n-dimensional random variable, independent of X with covariance matrix Ω. Show that the covariance matrix for $X + Y$ is $\Sigma + \Omega$. (If you have problems with the general situation, we will give full point if you show it for 2-dimensional case.)

4. (24% of final 2005) Assume that Z is a normally distributed random variable with variance 9 and mean 2, and that Z is independent of (X, Y) where (X, Y) is a bivariate normally distributed random variable with mean $\mu' = (0, 0)$ and variance-covariance matrix

$$\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

a) What is the conditional mean of $Y|X$?
b) What is the conditional variance of (X, Z) given Y?
c) What is the conditional mean of X given (Y, Z)?
d) What is the distribution of $2X^2 - 2XY + Y^2$?