Homework 7. Monday October 22, 2007. Due Monday October 29.

1. $(35 \%$ of Midterm 2, 2006) Consider two random variables $X$ and $Y$. Assume they both are discrete and that X can take the values 1 and 2 while Y can take the values 1,2 , and 3 . The probabilities for ( $\mathrm{X}, \mathrm{Y}$ ) are shown in the following table:

$$
\begin{array}{lll} 
& \mathrm{X}=1 & \mathrm{X}=2 \\
\mathrm{Y}=1 & 0 / 12 & 3 / 12 \\
\mathrm{Y}=2 & 2 / 12 & 1 / 12 \\
\mathrm{Y}=3 & 2 / 12 & 4 / 12
\end{array}
$$

i) Find the marginal distribution of X .
ii) Find the mean and the variance of X .
iii) Are the random variables X and Y independent?
iv) Find the probability $P(\{X>1\} \cap\{Y \leq 2\})$.
v) Find the conditional distribution of $X$ given $Y=1$.
vi) Find $E(X \mid Y)$ for $Y=1$.
vii) Find the distribution of $Z=X^{2} Y$. (We may not get to the formula for transformations on Wednesday, but give it a try. You to start by finding the support of $Z$.)
2. ( $40 \%$ of midterm 2, 2005) Consider two random variables $X$ and $Y$. Assume they both are discrete and that X can take the values 1 and 2 while Y can take the values 1,2 , and 3 . The probabilities for $(\mathrm{X}, \mathrm{Y})$ are shown in the following table:

$$
\begin{array}{lll} 
& \mathrm{X}=1 & \mathrm{X}=2 \\
\mathrm{Y}=1 & 1 / 12 & 2 / 12 \\
\mathrm{Y}=2 & 2 / 12 & 1 / 12 \\
\mathrm{Y}=3 & 2 / 12 & 4 / 12
\end{array}
$$

i) Find the marginal distribution of Y.
ii) Find the mean and the variance of $Y$.
iii) Are the events $\mathrm{X}=1$ and $\mathrm{Y}=1$ independent events?
iv) Are the random variables X and Y independent?
v) Find the probability $P(\{X>1\} \cup\{Y \leq 2\})$.
vi) Find the conditional distribution of $X$ given $Y=2$.
vii) Find $E(X \mid Y)$ for all $Y$.
viii) Find $E(X)$ using your answer to question vii). (If you don't use that answer, you will not get points. If you couldn't answer vii) you can make up the answer to that question for use in this one.)
ix) Find $\operatorname{Var}(X \mid Y=2)$.
x) Find the marginal distribution of X .

