Bent E. Sørensen

ECONOMICS 6331 – Probability and Statistics, Fall 2006

Homework 7. Wednesday October 25, 2005. Due Monday October 30.

1. Ramanathan, Practice Problem 5.10, page 99.

2. Let X and Y be normally distributed variables with means μ_x and μ_Y, resp., and variances σ_X^2 and σ_Y^2, resp.

a) Show that the random variable

$Z = X + Y,$

is normally distributed and find its mean and variance. (Hint: Find the Moment Generating Function. Use the law of iterated expectations.)

b) Argue, using the result in part a), that if $X_1, X_2, ..., X_n$ are normally distributed random variables with means $\mu_1, ..., \mu_n$, and $a_1, a_2, ..., a_n$ are constants then $a_1 X_1 + a_2 X_2 + ... + a_n X_n$ is a normally distributed random variable and state its mean and variance.

c) What is the distribution of the mean $\overline{X} = \frac{1}{n} \Sigma_{i=1}^n X_i$?

3. If

$$\Sigma = \begin{pmatrix} 20 & 10 \\ 10 & 10 \end{pmatrix}$$

verify that

$$\Sigma^{1/2} = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$$

Also find $\Sigma^{-0/5}$ and Σ^{-1} and verify that that $\Sigma^{-0.5'} \Sigma^{-0.5} = \Sigma^{-1}$.

4. Write the bivariate Normal distribution using the matrix notation for the multivariate normal. First show that the formula agrees with the formula for the usual univariate Normal if $N = 1$. Then, by inverting the variance matrix etc., verify that the formula for the multivariate Normal is identical to the formula for the bi-variate Normal distribution when $N = 2$. (What I want you to do is to start from the matrix notation and then derive the formula involving variances, correlation coefficient, etc. that doesn't involve matrices or vectors.)

5. Ramanathan, Exercise 5.7, page 118.