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ECONOMICS 6331 – Probability and Statistics, Fall 2005

Homework 7. Wednesday November 16, 2005. Due Monday November 21.

1. Ramanathan, Practice Problem 5.10, page 99.

2. Let X and Y be normally distributed variables with means μ_x and μ_Y, resp., and variances σ_X^2 and σ_Y^2, resp.

a) Show that the random variable

$Z = X + Y,$

is normally distributed and find its mean and variance. (Hint: Derive the moment generating function, use that the joint density of X and Y can be written as the product of the conditional density of X given Y and the marginal density of Y.)

b) Argue, using the result in part a), that if $X_1, X_2, ..., X_n$ are random variables with means $\mu_1, ..., \mu_n$, and $a_1, a_2, ..., a_n$ are constants then $a_1 X_1 + a_2 X_2 + ... + a_n X_n$ is a normally distributed random variable and state its mean and variance.

c) What is the distribution of the mean $\overline{X} = \frac{1}{n}\Sigma_{i=1}^n X_i$?

3. If

$$\Sigma = \begin{pmatrix} 20 & 10 \\ 10 & 10 \end{pmatrix}$$

verify that

$$\Sigma^{1/2} = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$$

Also find $\Sigma^{-0/5}$ and Σ^{-1} and verify that that $\Sigma^{-0.5'}\,\Sigma^{-0.5} = \Sigma^{-1}$.

4. Ramanathan, Exercise 5.7, page 118.