

ECONOMICS 6331 – Probability and Statistics, Fall 2004

Homework 7. Wednesday October 27, 2004. Due Monday November 1.

1. Ramanathan, Practice Problem 5.10, page 99.

2. Let X and Y be normally distributed variables with means μ_x and μ_Y , resp., and variances σ_X^2 and σ_Y^2 , resp.

a) Show that the random variable

$$Z = X + Y,$$

is normally distributed and find its mean and variance. (Hint: Derive the moment generating function, use that the joint density of X and Y can be written as the product of the conditional density of X given Y and the marginal density of Y .)

b) Argue, using the result in part a), that if X_1, X_2, \dots, X_n are random variables with means μ_1, \dots, μ_n , and a_1, a_2, \dots, a_n are constants then $a_1 X_1 + a_2 X_2 + \dots + a_n X_n$ is a normally distributed random variable and state its mean and variance.

c) What is the distribution of the mean $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$?

3. If

$$\Sigma = \begin{pmatrix} 20 & 10 \\ 10 & 10 \end{pmatrix}$$

verify that

$$\Sigma^{1/2} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$$

Also find $\Sigma^{-0/5}$ and Σ^{-1} and verify that that $\Sigma^{-0/5} \Sigma^{-0.5} = \Sigma^{-1}$.