Bent E. Sørensen

ECONOMICS 6331 – Probability and Statistics, Fall 2008

Homework 6. Wednesday November 5, 2007, due Wednesday November 12.

1. Assume that $X_1, X_2, ..., X_n$ are random variables with means $\mu_1, ..., \mu_n$, and $a_1, a_2, ..., a_n$ are constants. Use the formulas for multivariate means of variances of linear functions.
a) Find the mean of $W = a_1 X_1 + a_2 X_2 + ... + a_n X_n$.
b) Find the variance of W.
c) Find the mean and variance of $\overline{X} = \frac{1}{n} \Sigma_{i=1}^{n} X_i$?

2. Derive the formula for the bivariate Normal distribution from the matrix formula for the multivariate normal. First show that the formula agrees with the formula for the usual univariate Normal if $N = 1$. Then, by inverting the variance matrix etc., verify that the formula for the multivariate Normal is identical to the formula for the bi-variate Normal distribution when $N = 2$. (What I want you to do is to start from the matrix notation and then derive the formula involving variances, correlation coefficient, etc. that doesn't involve matrices or vectors.)

3. If

$$\Sigma = \begin{pmatrix} 20 & 10 \\ 10 & 10 \end{pmatrix}$$

and

$$\Sigma^{1/2} = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$$

verify that $\Sigma = \Sigma^{1/2} \Sigma^{1/2'}$. Also find the inverse of $\Sigma^{1/2}$ which we refer to as $\Sigma^{-0.5}$ and Σ^{-1} and verify that that $\Sigma^{-0.5'} \Sigma^{-0.5} = \Sigma^{-1}$.

4. Ramanathan, Exercise 5.7, page 118.

Please turn over.

5. Let

$$\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 4 \end{pmatrix}$$

be the variance matrix of a vector X where $X = (X_1\,,\, X_2)'$ and $E\,X' = (1,4)$. Let

$a = (2\,,\,3)'$ and

$$B = \begin{pmatrix} 1 & 4 \\ 8 & 0 \end{pmatrix}.$$

If $Y = a + B\,X$, what is $E\,Y$ and the variance of Y?