

**ECONOMICS 6331 – Probability and Statistics, Fall 2008**

Homework 5. Monday October 28, 2008. To do at session with Wided on Friday October 31st.

1. Ramanathan, Practice Problem 5.9, page 99.
2. Ramanathan, Practice Problem 5.10, page 99.
3. Let  $X$  and  $Y$  be normally distributed variables with means  $\mu_x$  and  $\mu_y$ , resp., and variances  $\sigma_x^2$  and  $\sigma_y^2$ , resp.

a) Show that the random variable

$$Z = aX + bY,$$

is normally distributed and find its mean and variance. (Hint: Find the Moment Generating Function. Use the law of iterated expectations. [Note, you would in general have to use the transformation formula, but the hint gives an easy way that works for the normal distribution].)

b) Argue, using the result in part a), that if  $X_1, X_2, \dots, X_n$  are normally distributed random variables with means  $\mu_1, \dots, \mu_n$ , and  $a_1, a_2, \dots, a_n$  are constants then  $a_1 X_1 + a_2 X_2 + \dots + a_n X_n$  is a normally distributed random variable and state its mean and variance.

c) What is the distribution of the mean  $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ ?

4. Ramanathan, Exercise 5.5, page 118.