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ECONOMICS 6331 – Probability and Statistics, Fall 2008

Homework 5. Wednesday October 15. Due Wednesday October 22.

1. Practice problem 5.1 in Ramanathan, p. 84.

2. Assume you roll two dice. Let X be the number of times you observe 1 or 3 eyes and let Y be the number of times you observe a 3. Derive
a) the joint probability distribution $f(x, y)$ (as in example 5.1).
b) $f_X(x)$, the marginal probability function for X.
c) $f_Y(y)$, the marginal probability function for Y.
d) $P(X < Y)$.
e) $P(Y = 2X)$.
f) $P(X + Y = 2)$. (We will soon cover how to do this soon more systematically, but for now you should find the probability of the set of (X, Y) pairs that sum to 2.)
g) Are X and Y independent or dependent?

3. Let $f(x, y) = (3/16)\, xy^2$; $0 < x < 2\,,\ 0 < y < 2$, be the joint density function for X and Y. Find the marginal density functions $f_X(x)$ and $f_Y(y)$. Find the distribution function (CDF) for X. Are the two random variables independent?

4. Let $f(x, y) = 1/6\, e^{-x/2-y/3}$ be the joint density function for X and Y. Find the marginal density functions $f_X(x)$ and $f_Y(y)$. Are the two random variables independent?

5. Consider two random variables X and Y. Assume they both are discrete and that X can take the values 1,2, and 4 while Y takes the values 0 and 2. The probabilities for (X,Y) are shown in the following table:

	X=1	X=2	X=4
Y=0	3/24	3/24	6/24
Y=2	3/24	5/24	4/24

i) Find the marginal probabilities of X and Y. Mark clearly which are the marginal probabilities of X and which are the marginal probabilities of Y. Explain what the marginal probabilities measure.
ii) Find the means and the variances of X and Y.

iii) Are the events X = 1 and Y = 2 independent events?
iv) Are the random variables X and Y independent?
v) Find the probability $P(\{X > 1\} \bigcap \{Y \leq 1\})$
vi) Find the conditional distribution of X given $Y = 2$.
vii) Find the random variable $E(X|Y)$.
viii) Take the mean of the random variable that you derived in vii) and verify that it equals $E(X)$.

6. Ramanathan, Practise problem 5.2, page 84.