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ECONOMICS 6331 – Probability and Statistics, Fall 2008

Homework 3. Monday September 22 (extended 9/26), 2007. Due Monday September 29.

1. (From Midterm 1, Spring 2004, counted 20%) Suppose we have some observations of Texans and Californians. The probability of observing a Texan is 1/3 and the probability of observing a Californian is 2/3. Now assume the following (made up numbers), namely that the probability that a Texan is a republican is 40% (so the probability that he is a democrat is 60%, we assume), and the probability that a Californian is a republican is 50% (so the probability that a Californian is a democrat is also 50%).
a) If you select one person from the population according to these probabilities, what is the probability that you will observe a republican from Texas? (Explain how you arrive at you answer)
b) In the model described for Californians and Texans, are the events A: {A person is a democrat} and the event B: {A person is from California} independent events? (Explain how you find the answer).
c) If you select 5 people randomly from the Texans. What is the expected number of republicans?

2. Assume that a random variable X is uniformly distributed on the interval $[1, 10[$.
a) What is the probability that $X < 3$? And the probability that $X > 5$?
b) What is the probability that $10 + 3X \geq 16$?
c) If $f(x) = 7 + 3\,x$, what is the density for the random variable $Y = f(X)$?
d) What is the distribution function (CDF) for Y in the previous sub-question?
e) If $f(x) = e^{2x}$, what is the density and distribution function of the random variable $Y = f(X)$?
You have to be explicit about the support (the area where the density for Y non-zero).

3. Ramanthan, Practice Problem 3.7.

4. Ramanthan, Exercise 3.10.

5. (28% of Midterm 1, 2005) Consider a uniform distribution on the closed interval $[0\,,\,1]$. Assume a random variable X follows this distribution.
a) Find the mean of X.

b) Find the distribution of $Y = \log(X)$. (Be specific about all details of the distribution.)
c) Find $P(Y < -0.5)$.
d) Find $E(Y)$.

6. Ramanathan, Exercise 3.15, page 59. Find the Moment Generation Function and use that to find the mean and variance.

7. If X is a Binomially distributed random variable with $p = 0.7$ and $n = 3$, what is the mean and variance of X? Find the answer two ways: a) directly summing over the outcomes; and b) using the moment generation function.

The following are good practice questions. I suggest you do those before the midterm.

P1. Assume that X is uniformly distributed on the interval $[-10, 10]$. Let $h(x)$ be the function x^2. Find $P\{h(X) \geq b\}$ (easiest to just use a graph) and $Eh(X)$ and verify that $Eh(X) \geq b\, P\{h(X) \geq b\}$ for $b = 2$ and $b = 6$. Also try to do the exercise for $h(x) = \exp(x)$.

P2. Show that if X is uniformly distributed on the interval $[0, 1]$ then $Y = -\theta \log(X)$ follows an exponential distribution with mean θ. Explain why Jensen's inequality implies that $E(Y) > \log(2)$ for $\theta = 1$.