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**ECONOMICS 6331 – Probability and Statistics, Fall 2007**

Homework 3. Wednesday September 12, 2007. Due Monday September 24.

1. (From Midterm 1, Spring 2004, counted 20%) Suppose we have some observations of Texans and Californians. The probability of observing a Texan is  $1/3$  and the probability of observing a Californian is  $2/3$ . Now assume the following (made up numbers), namely that the probability that a Texan is a republican is 40% (so the probability that he is a democrat is 60%, we assume), and the probability that a Californian is a republican is 50% (so the probability that a Californian is a democrat is also 50%).

a) If you select one person from the population according to these probabilities, what is the probability that you will observe a republican from Texas? (Explain how you arrive at your answer)

b) In the model described for Californians and Texans, are the events A: {A person is a democrat} and the event B: {A person is from California} independent events? (Explain how you find the answer).

c) If you select 5 people randomly from the Texans. What is the expected number of republicans?

2. Assume that a random variable  $X$  is uniformly distributed on the interval  $[1, 9[$ .

a) What is the probability that  $X < 3$ ? And the probability that  $X > 5$ ?

b) What is the probability that  $10 + 3X \geq 16$ ?

c) If  $f(x) = 7 + 3x$ , what is the density for the random variable  $Y = f(X)$ ?

d) What is the distribution function (CDF) for  $Y$  in the previous sub-question?

e) If  $f(x) = e^{2x}$ , what is the density and distribution function of the random variable  $Y = f(X)$ ?

You have to be explicit about the support (the area where the density for  $Y$  non-zero).

3. Ramanathan, Practice Problem 3.7.

4. Ramanathan, Exercise 3.10.

5. (28% of Midterm 1, 2005) Consider a uniform distribution on the closed interval  $[0, 1]$ . Assume a random variable  $X$  follows this distribution.

a) Find the mean of  $X$ .

- b) Find the distribution of  $Y = \log(X)$ . (Be specific about all details of the distribution.)
- c) Find  $P(Y < -0.5)$ .
- d) Find  $E(Y)$ .