

ECONOMICS 6331 – Probability and Statistics, Fall 2007

Homework 11. Monday November 19, 2007. For class Monday November 26 (we also want to talk about past finals during that class, so be prepared to ask questions about anything you find unclear, I may not talk about exercises that no-one asks about).

1. Ramanathan, Exercise 5.22, page 121.

2. a) If X is an $n \times k$ matrix of rank k (with $k \leq n$), verify that $M = X(X'X)^{-1}X'$ is an idempotent matrix and that $(I - M)$ is idempotent.

b) Let

$$X = \begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{pmatrix}$$

and find M .

c) What are the ranks of M and $(I - M)$?

3. (12% of 2004 final) Assume $X \sim \chi^2(9)$.

a) What is $E(X)$?

b) Derive the formula for the variance of a $\chi^2(k)$ (chi-square with k degrees of freedom) random variable.

4. Let

$$\Sigma = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$$

Find $\Sigma^{-1/2}$ the way I did in class Monday *or* using the formula for the conditional mean *or* finding it directly without thinking about random variables.