

Final Exam, April 29 —2 pages, 5 questions. All sub-questions carry equal weight.

1. (20%) Consider the AR(2) model

$$y_t = 3 + 0.2y_{t-1} + 0.2y_{t-2} + e_t ,$$

where e_t is white noise with variance 1.

a) Assuming the process is stationary, find the mean and variance of y_t .

Now let x_t be stationary process

$$x_t = 0.2x_{t-1} + e_t ,$$

(Notice, that the innovation to x_t is the same as the innovation to y_t).

b) Write down x_t as a function of y_t and lags of y_t .

2. (30%) Consider the case of the 2 agents, Jones (J) and Smith (S) who live for 2 periods in a 2 states-of-the-world model where agents can trade using a full set of Arrow securities. Goods can not be stored. Assume each of the agents have utility functions

$$\ln(C_0) + E_0 \ln(C_1)$$

The following table gives the possible endowments and the probabilities for Jones (J) and Smith (S):

	Smith		Jones	
State of the world:	A	B	A	B
period 0 endowment	20	20	20	20
period 1 endowment	10	30	30	10
Period 0 probability:	.5	.5	.5	.5

Assume that Jones and Smith are the only two agents in the world.

a) Assume that Jones and Smith can only trade in bond. Find the amount traded and the interest rate.

b) Assume for this question, that there is a full set of Arrow securities. Find the rate of interest between period 0 and period 1.

c) Find agents' consumption in each period and state of the world.

Now assume that Jones has the utility function

$$C_0 - \frac{1}{200}C_0^2 + E_0\{C_1 - \frac{1}{200}C_1^2\}$$

while Smith's utility function is unchanged. Assume there is still a full set of Arrow securities.

d) Assume again the agents only have access to a bond. Is the interest rate now higher than in part a? (Argue using logic.)

e) Will Jones now purchase a non-zero amount of the bond? Argue using logic (if you just say "non-zero," you won't get points, you have to give the sign and argue for it.)

3. (20%) Derive the general Euler equation. Make sure to state *all* assumptions needed. For full points, you need to state the exact assumptions, not just a sufficient set of assumptions.

4. (15%) Assume that the PIH holds and that the discount rate is 10 percent (and therefore also the rate of interest is 10 percent). Assume that an agent initial assets of 0 and has income in period 0 and in all future periods which satisfies the relation $Y_t = 10 + 0.7^t + w_t$, where

$$(1 - 0.3L)w_t = u_t ,$$

where $w_0 = 50$ and u_t is white noise.

Find the agent's consumption C_0 .

5. (15%) Consider the CAPM-model. Assume the safe rate of interest is 3%, the mean return to the market portfolio is 5% and the variance of the return to the market portfolio is 0.04. Now consider assets A and B. For these we know the distribution of the pay-outs. For A the payout is normally distributed with mean 100 and variance 10, while for B the payout is log normally distributed with mean 1000 and variance 5. Assume the covariance of the payout to asset A with the market return is 1 while the covariance of payout to asset B with the market return is 2. What would be the prices of assets A and B?